# Review of digital emulation of vacuum-tube audio amplifiers and recent advances in related virtual analog models

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**Abstract.** Although semiconductor devices have progressively replaced vacuum tubes in nearly all applications, vacuum-tube amplifiers are still in use by professional musicians due to their tonal characteristics. Over the years, many different techniques have been proposed with the goal of reproducing the timbral characteristics of these circuits. This paper presents a review on the methodologies that have been used to emulate tube circuits over the last 30 years for musical applications. The first part of the paper introduces the basic principles of tube circuits, with a common cathode triode example. The remainder of the paper reviews the tube sound simulation devices. The first of these emulations used analog operational amplifier circuits with the negative feedback designed to reproduce tube transfer. As DSP became more popular over the last decades for audio applications, efforts towards digital tube circuit simulation algorithms were initiated. Simulation of these devices are basically divided into linear models with digital filters that correspond to IIR analog filters and nonlinear digital models that corresponds to the tube circuit itself. The simulation of the first is straightforward, normally accomplished by the use of FIR digital filters. The last can be either accomplished approximation equations, that are known as digital waveshapers and their variants or by circuit derived techniques, such as the resolution of circuit ordinary differential equations solvers. Wave digital filter models are also variants of circuit simulation techniques that are also treated in this paper. The circuit derived techniques yield more precise simulations over the waveshapers but are always computationally more expensive so that a compromise between accuracy and efficiency is needed for real-time simulation of these devices.

**Keywords:** Virtual Analog Digital Models, Digital Audio Signal Processing, Vacuum Tubes, Electric Guitar, Applied Computing.

(Received October 15th, 2012 / Accepted June 26th, 2013)

## 1 Introduction

One of the elements that characterizes the electric guitar sound is the amplifier that the guitar is connected to. The first amplifiers were made up of electron tubes or valves (European name), which were the electronic active devices that dominated the industry up to the 1970's. Although the solid-state technologies have progressively replaced the vacuum tubes in most applications, these are still widely used in the electric guitar amplification. As a matter of fact, most of the guitar players prefer the valve amplifiers for their “warm” and “soft” sound. However, vacuum-tube amplifiers — such as the one shown in Figure 1 — are heavier, larger, less durable and more expensive than transistor amplifiers. It is also worth noting that since vacuum-tubes are mounted inside glass envelopes they are physically less robust than transistors, making the equipments that use these devices more fragile and delicate to handle. This has motivated the development of devices that aim to emulate the timbral characteristics of vacuum-tube audio amplifiers. Because the interest in these devices is closely related to their non-linear behavior when overdriven, there are no simple models that are able to accurately reproduce the tube amplifier sound. It is also worth noting that some other music related recording equipment such as compressors, mixing desks and preamplifiers still use vacuum-tubes in their circuitry because of its sound characteristics.

**Figure 1:** All tube: Fender super twin amplifier.

Since the modeling of vacuum-tube amplifiers and analog effects possesses its own challenges, this research field has been called “Virtual Analog” [41]. One of the main advantages of virtual analog models is that many different amplifiers can be simulated in the same DSP (Digital Signal Processing) system by adjusting the simulation parameters. This paper aims to provide a comprehensive and self-contained review of the digital techniques that have been used to emulate vacuum-tube amplifiers, such as those based on waveshapers and circuit simulation. The article is organized in the following topics:

- Description of the working principles of vacuum-tube amplifiers;
- Emulation of the tube amplifier sound by using solid state technology;
- Computer simulation of the linear behavior of vacuum-tube amplifiers;
- Computer simulation of the non-linear behavior of vacuum-tube amplifiers;
- Techniques for model validation;
- Concluding remarks.

## 2 Principles of vacuum-tube amplifiers

The first electronic devices were based on vacuum tubes instead of the solid state technology used nowadays. Since the invention of the diode tube by John Fleming [15] and the triode (equivalent to the transistor) by Lee De Forest [16] in the early 1900’s, all the radio, telephone, television and telegraph technology relied on these devices up to the 1960’s, when the transistor became popular. In recent years, tubes are still in use in microwave technology and high power and frequency applications due to their handling characteristics in this working regime [49], and in the musical scenario due to their sound characteristics. The truth behind the differences of distortion characteristics of semiconductor and vacuum-tubes is on physical nature of theses devices and go beyond the scope of this work, but can be empirically observed by visual inspection of any electrical components Dada sheet. The curves for electron tubes differ a lot from these of bipolar, jfet, or mosfet transistors. The work of Barbour [3] describes many technical differences of both devices in audio amplification.

### 2.1 Triode amplifiers

The basic circuit for amplifying signals is the common-cathode amplifier. It is similar to a JFET (Junction Field Effect Transistor) transistor in common-source configuration [34]. The circuit is pictured in Figure 2. The electrodes in the triode are the grid, the plate (anode) and the cathode. The two circuit networks are the plate-cathode $V_{pk}$ circuit and the grid-cathode circuit $V_{gk}$. Similarly to the FET transistor working principle tubes are transconductance operated devices, that is, a major current is controlled by a variable voltage source.

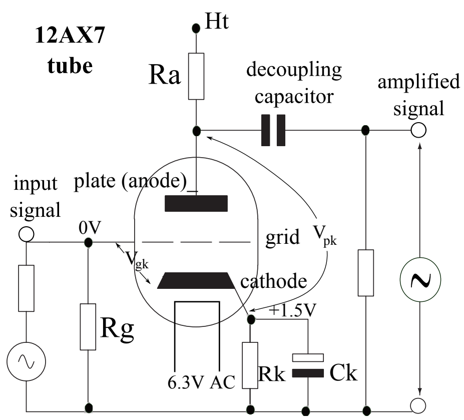


**Figure 2:** Common cathode circuit.

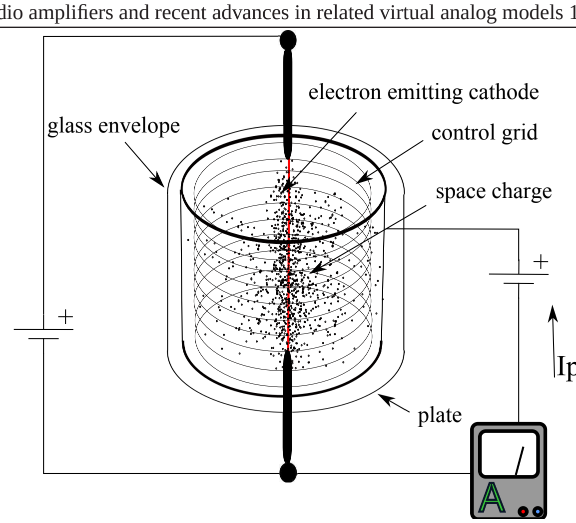


**Figure 3:** Electron flow inside tube.

In almost every audio circuit, the cathode $k$ is heated by a heater filament that is powered by a 6.3V AC power supply. The heating of the cathode metal surface accelerates the electrons. This makes electrons leave the metal surface creating a pool of electrons known as spatial charge, as observed in Figure 3. This process is known as electron emission, and is the working principle of every electron tube. As soon as cathode is negative (grounded) in relation to the plate, that is, at a high tension + 400V potential from ground, the emitted electrons from the cathode space charge are attracted to the plate by its high voltage electric field. This attraction forms an electric current inside the tube known as plate current. It is a small magnitude current in low power triodes, around 1 mA at idle. Similar to the FET transistors, the current flows in only one direction, that is, electrons flow from cathode to plate on vacuum tubes. Since the objective of the triode is to amplify alternating signals, the third element, the control grid, acts as the current control device inside the tube, it is physically positioned between the cathode and the anode. The control grid is negatively charged, so that it can limit the plate current, acting as a “gate” to electrons reducing the electron flow. Plate current will vary accordingly to the signal´s instant amplitude. As the grid becomes more negative, less current will flow until it ceases . This is called the cut off voltage for $V_{gk}$.

If an alternating current signal with $\omega_1$ frequency and with a negative DC voltage is applied to the control grid, the plate current ($i_p$) and will vary with the same frequency $\omega_1$. Since the plate resistor $R_a$ acts as voltage divider, from the plate and the power supply, consequently the AC signal in $v_{gk}$ creates a larger $v_{pk}$ AC signal with a $180°$ phase difference. The ratio of amplified signal in $v_{pk}$ over original input signal in $v_{gk}$ is represented by $A$ and $A = v_{pk}/v_{gk}$ for AC values. A typical triode amplification stage has an amplification factor A ranging from 40 to 65.

When the input signal is small, the output signal will be $v_{pk} = Av_{gk}$. If amplitude of the input signal is too large, the triode will reach its maximum peak to peak amplitude, which is limited by maximum plate current saturation and $V_{gk}$ cut off. When this situation occurs, the output waveform will be distorted and the tube will work on overdrive regime. The sound of the amplifier in these regimes are highly attractive to musicians, due to the fact that tubes when overdriven produce more odd harmonics and compression. Reproducing these nonlinear distortion characteristics accurately in DSP or analog circuits imposes many difficult technical challenges.

## 2.2 Pentode amplifiers

In most audio circuits, triodes are used in low power circuits for amplifying small amplitude signals. They are used in the first amplifying stages of the full amplifying chain until the signal has enough power to drive a loudspeaker. Usually two triode amplifiers in common cathode circuits are used before the power amplifying stage. In order to drive the loudspeaker higher power pentode (five electrodes) tubes are used. Pentodes are valves that were developed to overcame the problem of secondary emission that the previous tetrode tubes (four electrodes) had. The tetrode were developed to handle more power by adding and auxiliary grid at high DC voltage potential in order to increase their plate current flow. Secondary emission is the process that occurs in a part of the $V_{pk}$ domain range. If $V_{pk}$ is in the secondary emission domain range, electrons that accelerate violently towards the anode will hit it with enough velocity that other electrons will leave the plate surface and will be attracted back to the auxiliary grid which is also at high positive voltage potential, this reduces the plate current $I_p$ as the plate voltage $V_p$ increases. The domain where secondary emission occurs is also known as Tetrode kink [20] as it creates a valley for $I_p$ in a tetrode $V_p, I_p$ plot. The total suppression of the secondary emission was accomplished by addition of another grid, positioned between the auxiliary grid and the anode named suppression grid. This grid is normally at ground potential, in some tubes as the 6L6GC it is internally connected to the cathode. Although this famous tube works just like a pentode it is technically considered as an Electron Beam Tetrode.

Normally, pentodes are associated with more efficiency and higher power [20]. Common power pentodes for guitar amplifiers are the KT66,KT88, EL34, EL84, 6V6 and other less common tubes. The power handling section is called the power amplifier and varies from 5 W to 200 W in commercial amplifiers. Triode and pentode tubes have high impedance at both grid (M$\Omega$ range) and plate (k$\Omega$ range) circuits, whereas loudspeaker impedances range from 4 $\Omega$ to 16 $\Omega$. For that reason, power amp tubes are unable to drive a loudspeaker unless an energy conversion device such as a transformer is used. Therefore, a special audio frequency transformer is added in between the power tubes and the speaker to correctly match the different impedances. Such devices must be properly designed to couple audio frequencies as low as 20 Hz. The iron core introduces nonlinear effects such as iron core saturation and hysteresis. The transformer is also responsible for removal of DC components, so that only alternating current signals feed the loudspeakers.

More detailed information on tube circuit theory is present in classic electronics literature, such as the RCA Radiotron Handbook by Langford-Smith [26].

## 3 Tube emulation with solid state devices

Historically the first devices that attempted to substitute tubes were implemented to fit directly into the tube socket as a direct substitute. The first invention occurred in 1970 by Robert L. Eby [11], which included two 2N3393 transistors and a 1N4009 diode in order to substitute double triode tubes in Hartley oscillator circuits found in electronic organs. Another example of these devices is a circuit patented by Schneider & Burman in 1973 [46], which replaced a pentode preamplifier tube by an integrated chip containing two JFET transistors. This can be regarded as the first tube emulation device.

In the early 1980's, Peavey Electronic's engineer Sondermeyer patented an invention intended to simulate tube sound characteristics [47]. The developed circuit presented in this patent uses operational amplifiers with negative feedback networks in order to deliberately replicate tube amplifier transfer characteristics. In 1996, another patent [48] by the same company describes a circuit that is designed to replicate crossover distortion from pentode tube AB class power amplifiers, which is another type of nonlinear distortion presented in vacuum-tube amplifiers. This was accomplished by using class B biased operational amplifiers.

## 4 Tube amplifier linear digital models

As far as amplifier simulation is concerned, the models must be categorized as either linear or nonlinear since analog amplifiers have corresponding linear circuits (RC or LCR filters) and nonlinear circuit elements (transistors, operational amplifiers, diodes and electron tubes). Unlike nonlinear systems, a linear system is characterized by its impulse response and does not increase the bandwidth of the input signal.

From this point on, it is necessary to identify a circuit model as being linear or nonlinear. Digital models of linear circuits are mostly accomplished by the use of digital filters that recreate the frequency response of the corresponding analog filtering circuit. For digital models of nonlinear circuits, special functions must be used to generate the nonlinear transfer of these circuits. This section deals with linear models, the nonlinearities will be considered in the next section.

The linear part of amplifiers consists of the network of resistors, capacitors and potentiometers responsible for the frequency response of an individual amplifier, another name for that network is the *tonestack*. The tonestack is also responsible for reducing midrange frequency response that is originated from resonant midrange frequency from magnetic coil pickups from electric guitars [39]. One of the main characteristics that distinguishes one amplifier design from the another is the *tonestack* capacitors, resistors and potentiometer values. When knobs (potentiometers) are set to a desired frequency response by the user, this network corresponds to an IIR (infinite impulse response) filter. So, when the user alters knob settings of an amplifier, he or she is actually altering the impulse response of a linear system.

A way to digitally replicate the tonestack behavior is to use digital filters to recreate a desired impulse response corresponding of a knob setting. A patent by Fender Musical Instruments Corp. [8] describes a DSP device that recreates the analog filter responses. The basic idea is to have many different digital tonestacks from different amplifiers in the same device. In this patent, a data acquisition procedure was accomplished in order to obtain the frequency response of many different real vacuum-tube amplifiers at various knob settings. The mapping from knob settings to a corresponding filter coefficients is stored in a lookup table, therefore storage space is an issue for practical implementations. For that reason the patent provides a method for a 3D linear interpolation in order to obtain the filter coefficients of knobs setting values that were not present in the table. However this patent does not mention if the filters are implemented as FIR or IIR filters.

Yeh [55] simulated the tonestack of the famous Fender Bassman F5A amplifier by using the circuit equations in the state-space representation to derive the FIR digital filters. More recently, Dempwolf [9] used a variant of this approach to simulate the tonestack of a different Fender Bassman amplifier, the AA763 model.

In a patent, Gallien e Robertson [17] describes a system identification technique followed by a circuit analysis procedure to simulate linear circuit filters of vacuum-tube amplifiers. The first procedure uses a set of test signals as inputs in an analog filter to be simulated. Using this technique, the amplitude and phase responses of the analog system are measured for a given tonestack setting. Then, a software for circuit simulation is used to find a theoretical circuit that possesses the same amplitude and phase responses of the analog tonestack. After a digital circuit response is close enough to the original system response, the bi-linear Z-transform of the “virtual analog filter” is implemented as first order high pass and low pass FIR filters by which its coefficients are weighted in order to obtain the desired output. An alternative to these methods was developed by Gustafon et al. [19], in order to record the frequency response of an amplifier, its frequency response is obtained by a system identification approach. A patent by Gustafson et al.[19] uses a black box approach to obtain the filter coefficients of a “generic” Laplace transform $H(z;\theta)$ that can model different tonestacks and frequency responses once the parameters were obtained experimentally. In this process, the parameters are initially set randomly or by the developer’s previous experience. Experimental input-output data is obtained in order to determine theses parameters. A standard frequency system identification procedure is then utilized to fine tune these parameters to match a desired frequency response. It is also worth noting that since ten different knob settings in each of the four potentiometer leads to $10^4$ different frequency responses, so interpolation techniques for the filter coefficients is also necessary for practical implementation of this simulation.

## 5 Tube amplifier nonlinear digital models

### 5.1 Static waveshaper functions

The most straightforward way to generate nonlinear distortion in digital audio signals is by applying a nonlinear function into each sample of the signal, as illustrated in Figure 4. These functions can be of many types and are known as static waveshapers, where the term “static” is due to the fact that such waveshapers are memoryless systems. Waveshaping functions for audio applications were introduced by Arfib [2] and Le Brun [27] in the late 1970s.

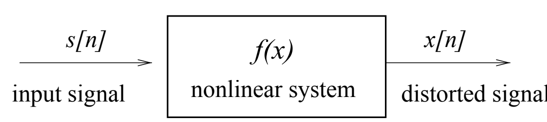


**Figure 4:** Nonlinear system representation.

Afterwards, Araya and Suyama [1], in a patent for the Yamaha Corporation, proposed in Eq. 1 as a waveshaper to simulate nonlinear tube distortion, which is illustrated in Figure 5. The motivation for the development of this waveshaping function was to provide a digital sound effector that was capable of providing a mild and soft distortion such as the ones of vacuum-tube amplifiers.

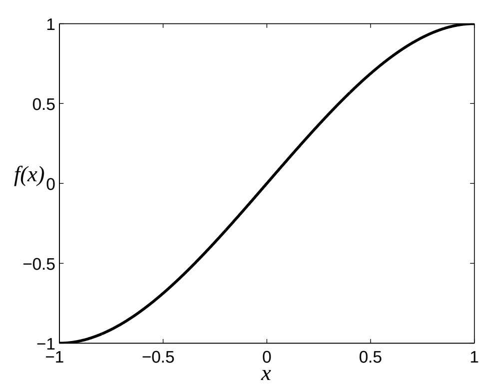


**Figure 5:** Waveshaper proposed by Araya and Suyama [1].

$$f(x) = \frac{3}{2}x\left(1 - \frac{x^2}{3}\right) \quad (1)$$

Figure 6 shows a 1 kHz sine wave distorted by such a waveshaper in the time and frequency domain, where the coefficients of the Fourier series (in dB) have been normalized for clarity purposes. As expected, the result of using a nonlinear function is an increase of the harmonic content of the output signal. However, it can be noticed that this waveshaper produces only a mild distortion. Therefore, the patent by Araya and Suyama [1] describes a DSP system in which the waveshaper is applied three times to the original signal in order to produce more distortion.

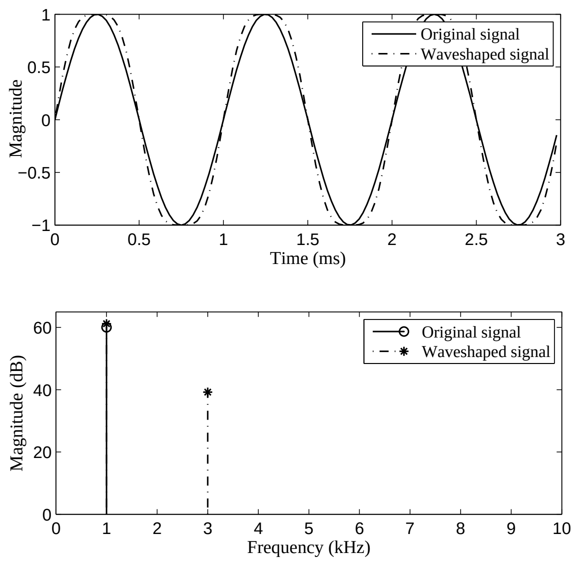


**Figure 6:** A 1 kHz sine wave filtered by Eq. 1 in the time and frequency domains.

Two years later, Doidic et al. [10] proposed in a patent the following two new nonlinear functions for static waveshaping, which also aim to reproduce tube distortion:

$$f(x) = \text{sign}(x)(2|x| - |x|^2) \tag{2}$$

and

$$f(x) = \begin{cases} -\frac{3}{4}\left\{1 - [1 - (|x| - 0.032847)]^{12} + \right. \\ \left. + \frac{1}{3}(|x| - 0.032847)\right\} + 0.01, \\ \qquad \text{if } -1 \le x < -0.08905 \\ -6.152x^2 + 3.9375x, \\ \qquad \text{if } -0.08905 \le x < 0.320018 \\ 0.630035, \\ \qquad \text{if } 0.320018 \le x \le 1 \end{cases} \tag{3}$$

The former is a symmetric function that intends to simulate the effects of a tube amplifier, in this function, **sign** is a signal operator that returns 1 if $x <$, 0 if $x = 0$ and -1 if $x < 0$. Equation 3 is an asymmetric function that aims to model several tube preamplifiers. It is worth noting that the original patent

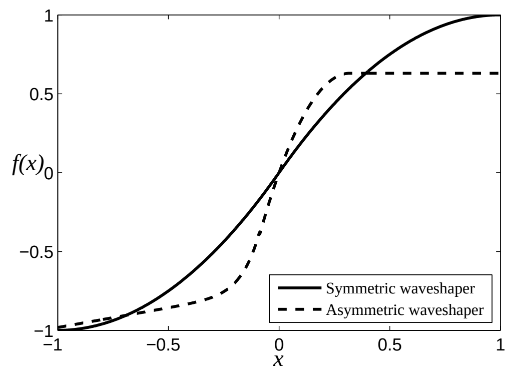


**Figure 7:** Symmetric and asymmetric waveshapers proposed by Doidic et al. [10].

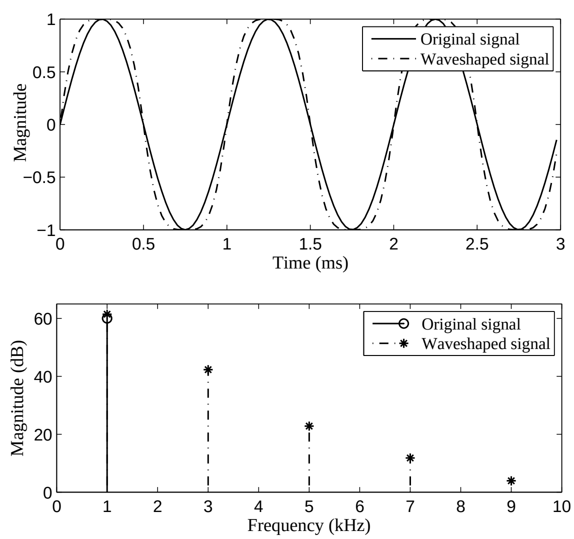


**Figure 8:** A 1 kHz sine wave filtered by Eq. 2 in the time and frequency domains.

has some typographical errors in the asymmetric waveshaper equation, so that it gives a plot that does not correspond to such an equation. Thus, Eq. 3 presented here is a version of the original patent corrected according to Pakarinen and Yeh [44]. Figure 7 shows the curves corresponding to the symmetric and asymmetric waveshapers, which match the original patent drawings.

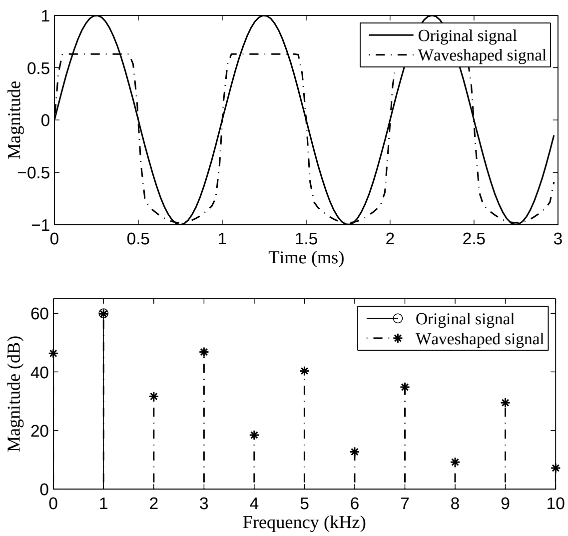


**Figure 9:** A 1 kHz sine wave filtered by Eq. 3 in the time and frequency domains.

Figures 8 and 9 show the distortions introduced by Eqs. 2 and 3 into a 1 kHz test tone, respectively. Notice that Eq. 3 produces more distortion than the other waveshapers and, due to its asymmetry, it introduces even harmonics in the spectrum of the test signal, as well as a DC component.

More recently, Gallo [18] proposed in a patent the following alternative waveshaper to emulate the effects of vacuum-tube amplifiers:

$$f(x) = \begin{cases} \dfrac{(k_1 + x)}{(k_2 - x)}, & \text{if } x < a \\ x, & \text{if } a \leq x \leq b \\ \dfrac{(x - k_3)}{(x + k_4)}, & \text{if } x > b \end{cases} \tag{4}$$

where $k_1 = a^2$, $k_2 = 1 + 2a$, $k_3 = b^2$ and $k_4 = 1 - 2b$. The values of $a$ and $b$ can be freely chosen between $-1.0$ and $+1.0$ in order to control the characteristics of the nonlinear function. Since these two parameters are independent of each other, the positive and negative values of the input signal can be treated separately, which helps mimic the behavior of real vacuum-tube amplifiers. Small signals in the range $a \leq x \leq b$ remain undistorted.

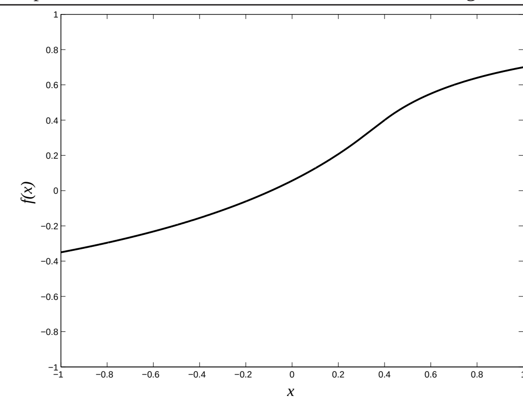


**Figure 10:** Asymmetric waveshaper proposed by Gallo [18]

The waveshaper of Eq. 4 is illustrated in Fig. 10, with parameters $a = 0.3$ and $b = 0.7$. These parameters were chosen to highlight the asymmetric characteristic of this function.

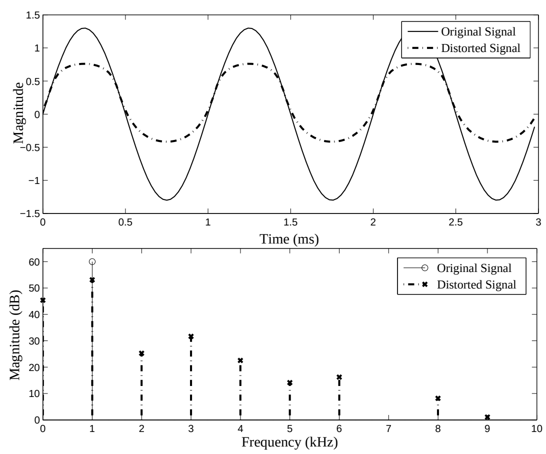


**Figure 11:** A 1 kHz sine wave filtered by Eq. 4 in the time and frequency domains with parameters $a = 0.3$ and $b = 0.7$.

Figure 11 exhibit the distortion introduced by Eq 4 into a 1 kHz test tone. It is worth noting that this asymmetric distortion introduces odd harmonics that have have larger magnitude than the even harmonics. Values $a$ and $b$ can be altered creating many different curves and distortion characteristics for Eq. 4.

The patent by Gallo [18] also describes other parameters that can be added to this function to increase the system versatility, which are not presented in this paper for the sake of clarity.

A patent by Gustafson et al. [19] uses Chebychev polynomials and a system identification approach to create a set of polynomial coefficients and functions that can be altered in real-time to simulate the dynamic non-linearities of vacuum-tube amplifier circuits. The polynomial coefficients were calculated by exiting a tube amplifier with various test signals. Once a set of coefficients is calculated in the system identification process, a set different polynomials is used in the simulation. A exchange among waveshaping funcions (polynomials) occurs in real time in order to simulate the tube's dynamic non-linearities, where a mode parameter selects which of static non-linear functions is to be active. This selector works by selecting a function according to the energy of the input signal in different audio bands.

### 5.2 Aliasing and upsampling

It is well known that nonlinear distortion expands the frequency bandwidth of the signal, as inspection of Figures 6, 8 and 9 reveals. In analog amplifiers, the frequencies of the new added harmonics are dictated by the amplifier nonlinear behavior and the frequency response of the tone-stack. The latter is a linear filter that enhances some frequencies and attenuates others. In digital amplifier models, this bandwidth expansion will lead to undesirable effects if the Nyquist sampling theorem is violated. Such a theorem states that, in order to properly sample a continuous-time signal, its maximum frequency must be smaller than a half of the sampling frequency [36]. Otherwise, aliasing distortion will take place, i.e., the introduced harmonics with frequencies larger than the maximum allowed frequency will fold back to the signal bandwidth as audible frequencies, leading to undesirable artifacts in tube simulation digital models. In cases of strong nonlinearities, this is more critical since the bandwidth is expanded in a more significant way.

This issue can be overcome by increasing the sampling rate of the sampled signal before the nonlinear processing block, which is a largely used technique called "upsampling" [36]. As a consequence, an interpolation rule must be used for creating sample values for adjacent samples. Clearly, upsampling leads to an increase of CPU time, and thus low cost DSP systems for tube emulation use lower sampling frequencies for the nonlinear processing. Figure 12 shows a block diagram of the system described in the patent by Doidic et al. [10], which illustrates the use of upsampling in digital tube simulation. The sampling rate of the discrete-time signal is increased by a factor of eight before the digital nonlinear processing. After the waveshaping, the signal is digitally low-pass filtered (with a digital antialiasing filter, not shown in the figure) in order to limit its bandwidth. Then, it is downsampled to the original sampling rate before applying the linear FX and the D/A conversion.

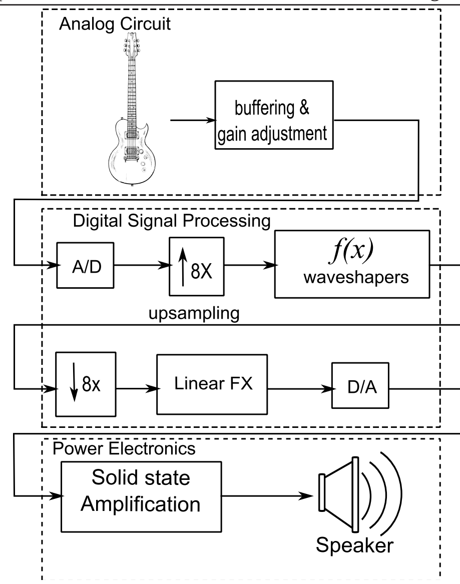


**Figure 12:** Block diagram of a DSP system for simulating vacuum-tube amplifiers [10].

### 5.3 Circuit derived waveshapers

The static waveshapers presented in Section 5.1 have been developed without taking into account the physical characteristics of the tube amplifier circuit. In a different approach, Bendiksen [4] used circuit information to obtain a static waveshaper for simulating the tube triode distortion, which is given below:

$$f(x) = \frac{x - Q}{1 - e^{-d(x-Q)}} + \frac{Q}{1 - e^{d(Q)}} \tag{5}$$

where $x$ is the input signal, $d$ is a distortion level parameter, $Q$ is the triode working point and $f(x)$ is the output signal. $Q$ is a circuit parameter on every amplifying circuit, which corresponds to the grid bias voltage in relation to the cathode. As $Q$ increases, the system approaches to the plate current saturation, so that more distortion occurs. On the other hand, as $Q$ assumes a large negative value, the system presents an almost linear behavior.

This waveshaper typically generates an asymmetric distortion, which normally occurs on triode tubes when overdriven. However, no distortion will take place if the input signal is of low amplitude, so that $f'(x) \approx 1$. Bendiksen [4] also proposes the use of a one-pole low-pass filter to simulate the triode Miller capacitance, and a high-pass filter to attenuate the DC component introduced by the asymmetric waveshaper.

Equation 5 yields very different outputs depending on the waveshaper parameters. Therefore, for illustration purposes only, Figure 13 shows a 1 kHz sine wave filtered by Eq. 5 with $Q = -1.1$ and $d = 1.0$. Before waveshaping the input signal, a gain of 10 has been applied to it. As expected, inspection of Figure 13 reveals that the spectrum of the output signal contains even and odd harmonics, as well as a DC component.

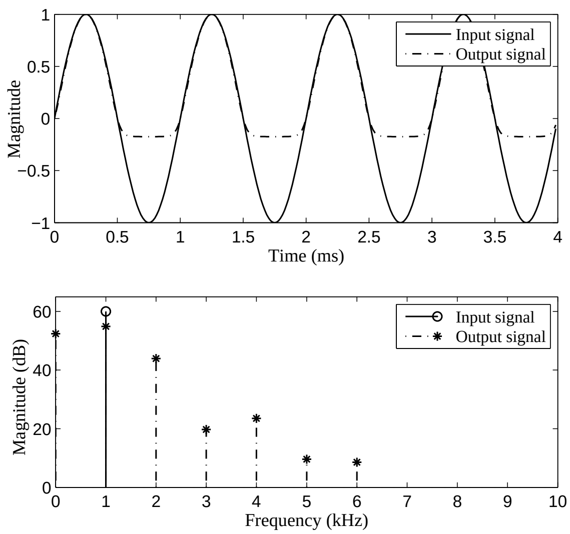


**Figure 13:** A 1 kHz sine wave filtered by Eq. 5 in the time and frequency domains.

### 5.4 SPICE models

SPICE (Simulation Program with Integrated Circuit Emphasis) is a circuit simulation software that can be used in vacuum tube modeling. It is based on transient Modified Nodal Analysis (MNA), with nonlinear equations to simulate circuit elements such as diodes, transistors and vacuum tubes, and differential equations for linear components modeling, such as capacitors and inductors. The circuit equations are written in the form of $\mathbf{I} = \mathbf{GV}$, where $\mathbf{V}$ is a vector containing the voltages associated to the circuit nodes, $\mathbf{I}$ is a current vector, and $\mathbf{G}$ is the conductance matrix. $\mathbf{G}$ is typically a sparse matrix because the circuit components are usually connected just to a few other components. To solve such a system of linear equations, LU decomposition is used. The complexity of such algorithm is $O(N^3)$, where $N$ is the order of the square matrix $\mathbf{G}$ . In the case of a typical circuit simulation, the nodal MNA analyzes results in sparse matrices, with the reduction of complexity down to $O(N^{1.4})$ according to Pakarinen et al.[43].

In the past, SPICE was used only for solid state electronics simulation and design. After the tube amplifier comeback in the late 1990's [3], a sequence of models were proposed and used as tools for designing and validating digital tube amplifier models. In 1995, Rydel [45] proposed the first vacuum-tube models for SPICE, just a few months before Leach [28] proposed similar models for the triode and pentode tube equations for the simulation of amplifier circuits, these can be considered as the first circuit informed simulation methods. However, circuit simulation in softwares such as SPICE require a significant CPU time, which makes their use unsuitable for real-time audio applications.

The Koren's [25] equations are improvements over the above mentioned Leach's equations, and are used in SPICE simulations of amplifiers or in vacuum tube circuit informed simulations for both triode and pentode models. These equations are considered as phenomenological equations since they do not follow well-established physical laws and were obtained experimentally. More information on Vacuum-tube models and comparison of different models is preset in the work of Cohen and Hélie [6].

Novel experimental equations for triode tubes have been recently proposed by Cohen and Helié [7], which were obtaining through measurements of tube transfer characteristics, capacitances and other metrics in a DSpace DS2004 DSP hardware. Compared to the Koren's relations, these equations require additional fitting parameters and include the grid current in the formulation, but they do not apply to pentodes. These new equations exhibit potential for replacing Koren's triode equations. However, further studies should be conducted in order to check their ability in accurately mimicking the triode behavior.

### 5.5 State-space representation

State-space representation has been used in several nonlinear circuit simulations of analog effects and has shown potential for precise simulation of such effects [54, 52]. Normally, this approach is used together with a numerical method to solve ordinary differential equations (ODE), such as Newton-Raphson, Euler and Runge-Kutta. As far as guitar effects are concerned, Yeh et al. [54] present a comparison of these methods for numerical stability, precision, complexity and other addressed issues in solving nonlinear ordinary equations. In this representation, the system of equations is represented by Eq. 6 and Eq. 7.

$$\dot{x}(t) = \boldsymbol{A} \cdot x(t) + \boldsymbol{b} \cdot u(t) + \boldsymbol{C} i(v) \tag{6}$$

$$y(t) = \boldsymbol{d} \cdot x(t) + e \cdot u(t) + \boldsymbol{f} i(v) \tag{7}$$

where $x(t)$ and $\dot{x}(t)$ are state variables, $u(t)$ are inputs, $y(t)$ are outputs. $\boldsymbol{A}$ is the state matrix, $\boldsymbol{b}$ input column vector, $\boldsymbol{d}$ is the output row vector and $e$ is the feedthrough scalar value. The number of state variables is the number of energy storing circuit elements, for this type of circuit it corresponds to the number of capacitors in the circuit. The terms $\boldsymbol{C}i(v)$ and $\boldsymbol{f}i(v)$ are the non-linear equations derived from special vacuum-tube equations.

A special technique for creating nonlinear digital models from SPICE netlists and using the state space formulation for solving the ODEs is proposed by Yeh et al. [53]. The circuit is discretized by the use of the Discretized Kirchoff Nodal Analysis of the circuit, also known as Discretized K-method. A common-cathode 12AX7 preamp circuit was implemented and tested as a case study [51].

Cohen and Helie [5] report a digital simulation of a single ended class A power amp with a 6l6GC tube and output transformer, simulated by using the Koren's pentode equations. This work also makes use of the state-space representation to model the nonlinear ODEs. Some parameters of the power amplifier model are based on the datasheet of the Plitron PAT-3050-SE-02 audio transformer, which is modeled as a linear device, although real transformers have nonlinear effects such as iron core saturation and hysteresis. The numerical implicit Newton-Raphson method was used for solving the nonlinear system of equations for this circuit. As opposed to this work Macak [30] reports non-linear core models for an audio transformer model also using state-space formulations and numerical solution using Newton-Rahpson's method.

High gain amplifiers, such as the Mesa-Boogie Dual Rectifier and the Marshall JCM800, use many triodes in cascade for higher gain, which leads to higher distortion. In order to simulate the tone of an full vacuum-tube high-gain amplifier, many tube stages must be simulated, each with a varying circuit topology. Aiming to simulate one of these circuits, Macak [31] implemented a JCM800, a Mesa Boogie and a Fender Super Reverb using circuit equations based on Koren's models. The interaction between stages was modeled using a block-wise approach where the each two adjacent cascade stages are used to obtain a single processing block. For this methodology, consider amplifying stages one and two as examples, where the signal is amplified in this order. The plate circuit impedance of stage one and the grid circuit impedance of stage two are calculated as parallel impedances, this impedance is used for each processing block for tube circuit equations. Since the Marshall JCM800 has four amplifying cascade stages four "independent" processing blocks are needed to model the circuit. Three The circuit was modeled as a system of nonlinear ODEs solved using the Newton-Raphson method.

Other simulated circuits include the phase inverter circuit, and power amp pentode push pull amplifier, without the output transformer, replaced by a $R_a$ resistor as a load for the tube plate network. All circuit equations for the power amp use Koren's pentode equations. An improvement over the Marshall JCM800 preamplifier was accomplished also by Macak $\&$ Schimmel [32] where the equation results were also stored in a 3D precomputed matrix for real-time efficiency. Different interpolation methods were tested to compute values not stored in the matrix, where the spline approximation was considered the best method. A Fender stile pre-amplifier with two 7025 tubes and a tonestack was also modelled by Macak et al [33] using state space representation with the objective of reducing the pre-computed look-up table storage size for both non-linear and linear parts of the amplifying circuit. In this work, a cubic spline was also utilized as interpolation technique since $i_p(v_p, v_g)$. This type of optimization is necessary in commercial audio pluggins, as they need to be optimized in both memory space and CPU $\%$ usage as many plugins run multi-threaded in a audio recording software such as Pro Tools, Cubase or Sonar. The researches by Macak et al. are applied in the products of the Audiffex company located in Czech Republic.

### 5.6 Wave Digital Filters

Wave Digital Filters (WDF) is a special type of digital filter that have a valid interpretation in the real world. This means that the behavior of a physical and complex system can be modeled by this approach [50]. WDF have been used to create digital models of vacuum-tube amplifiers over the last few years [21, 22, 23, 42].

The main advantages of this model methodology are: high modularization potential, energy preservation by the use of Kirchoff laws and good numerical properties in its implementations, leading to efficient real time digital models of virtual analog circuits for audio effects.

WDF were originally developed to solve lumped electronic circuits, by creating digital circuit models from the original schematic. Each circuit element is modeled by its circuit equation. The interconnection among elements is accomplished by WDF ports (adapters), in the same way as the original circuit. These adapters can be series adapters or parallel adapters, transformers, dualizers, gyrators, and others.

The main characteristic of the WDF models is that the bidirectional interaction of circuit elements are considered. This is accomplished by the concept of wave scattering propagation. The WDF formalism is based on 'Voltage' wave notations, in Equation 8, $a$ corresponds to incoming wave and $b$ corresponds to the reflected wave, $V$ is the Voltage, $I$ stands for current in the same ways as Kirchoff variables and $R_{p1}$ and $R_{p2}$ are the port or reference resistances as pictured in Figure 14.

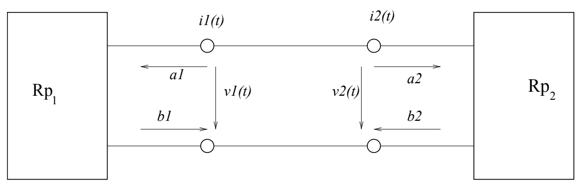


**Figure 14:** Wave Digital Filters Signal Propagation of Kirchoff Variables

$$\begin{cases} a = V + R_p I \\ b = V - R_p I \end{cases} \tag{8}$$

More details on WDF theory is covered by Fettweis [13].

The first WDF model of a triode common cathode amplifier such as that of Figure 2 was implemented by Matti Karjalainen and Jyri Pakarinen[22]. It models the circuit with simplicity. The non-linear tube characteristics is implemented by a non-linear resistor implemented using Koren´s triode tube equation. The WDF schematic of such model is illustrated in Figure 15.

An enhanced version of this triode was accomplished by the same authors [42], where secondary effects were added, such as blocking distortion and the Miller capacitance. The nonlinear processing of the triode was also accomplished using Koren's triode models.

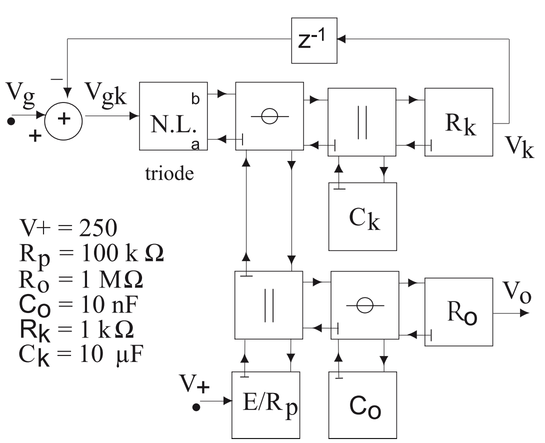


**Figure 15:** Symbolical Network representation of the first Wave Digital Filters triode models by Karjalainen and Pakarinen[24].

A particular digital WDF model for triodes was implemented for the Csound environment by Fink [14], where the classic comon-cathode stage circuit of Figure 2 was implemented. The C-sound opcode was compiled into C code for real-time efficiency. This work also implements the nonlinear behavior of Koren´s triode equations.

### 5.7 Output chain and audio transformer WDF models

One of the elements that also alters the sound characteristics of the sound in a tube amplifier is the output transformer that is placed in the power amplifier section, in is inserted between the power-amp tubes and the loudspeaker. Its intension is to provide the correct loading for the power tubes. This special transformer also add non-linear distortion to the amplifier due to its core saturation and hysteresis. Some digital simulations of its characteristics were accomplished by WDF developers.

A WDF simulation of a linear output transformer model with a triode amplifying power amplifier was reported by Pakarinen et al.[43]. This model does not include the nonlinear behavior of the output transformer but was accomplished with the use of parameters for the speaker and transformer of real devices.

A special Wave Digital Filter model of an output chain of a power amplifier with a KT88 pentode in triode connection in series with an audio transformer and speaker was accomplished by Paiva et al. [38]. In this WDF model, the nonlinearities of the transformer were introduced by the use of a gyrator and capacitor transformer model. The generated WDF model´s parameters were adjusted from measurements of a real Fender NSC041318 (in single ended configuration) and a Hammond T1750V. The transformer model was also validated using these two real transformers. The nonlinearities of real audio frequency transformers are derived from hysteresis and core saturation. Nevertheless the results from this research reveal that these effects were only noted in low frequencies, no push pull transformer power amplifier model was proposed, and leaves a gap to be filled in future researches in the 'Virtual Analog' field.

In spite of this, WDF methodology provides efficiency, modularity and exhibits itself as promising tool for modeling analog circuits it has however some shortcomings. A standard framework for the development of WDF models has not became apparent over the years and consequently models and blocks are not interchangeable among developers. Another drawback is that some circuit topologies do not map directly to WDF port connections.

## 6 Model Validation Techniques

When a digital model is implemented, the comparison of both the analog and the virtual analog models must be made since the Digital Model is always a simplified version of the analog circuit. Special signal processing techniques have been proposed in order to compare output of different systems. One special testing methodology was created by Pakarinen [40] in a special toolkit for Distortion of Signals. This methodology uses single tone (classic Fourier analysis) , intermodular [29], logsweep [12] and transient signal analysis [37] of a system´s output signal and also the input signal. These techniques were used in a case study by Oliveira et al. [35] to characterize nonlinear distortion of an all tube Giannini True Reverber amplifier designed by Carlos Alberto Lopes. Another common validation procedure is to use Spice simulation of circuits and compare both output signals. Often a musician´s trained ear is the best tool for judging either if a digital model performs its task in satisfactory way [44].

## 7 Concluding Remarks

Many models have been proposed to emulate the behavior of vacuum-tube audio amplifiers, with different degrees of complexity. However, the currently available models seem not able to accurately reproduce the sound quality of such devices. It can be noticed that the circuit simulation-based techniques are the most used approach, leading to good results. However, for real time simulations, there is always a trade-off between accuracy and computation time. Besides, more research should be conducted in order to derive a set of equations that will satisfactorily describe the physical behavior of tubes. Finally, it is worth emphasizing that there is a lack of subjective evaluations aiming to compare the sound quality of real and virtual vacuum-tube amplifiers. In this sense, psychoacoustic investigations would be helpful in validating the proposed models and to guide future works on this topic.

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