# Kalman-Takens filtering in communication systems

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**Abstract.** We apply a model-free predictor based on the Kalman Filter to signal-to-noise ratio (SNR) data from a mobile communication system experiment. The experiment consist of collecting performance indicators on a mobile device during the trajectory of a city bus. In particular, we analyze the SNR measured by the mobile, which is collected every second via an application. Since some mechanisms in a mobile network depend on the SNR, like power control and handoff processes, our results show that it is possible to use prediction models to improve several procedures in mobile communications systems.

**Keywords:** Prediction models, Mobile systems, Kalman filters.

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## 1 Introduction

Since the introduction of wide-band communications in mobile networks, the user base has experienced exponential growth. Currently, there are over 145 thousand operational commercial 5G networks [19].

This escalating demand has elevated resource allocation to a crucial research focus in mobile networks. Furthermore, given the real-time nature of bandwidth distribution, resolving resource issues through optimization techniques is generally impractical [26].

Significant attention has been directed towards predictive resource allocation, as forecasting specific network parameters holds the potential for more efficient resource distribution among subscribers. In a linear scheme applied to predict reference signal received power (RSRP) in Ref. [1], and a similar approach in Ref. [27], an optimization procedure was employed to predict signal-to-noise ratio (SNR) values, resulting in an enhancement compared to an round robin (RR) algorithm-based allocation method.

The application of the Kalman filter (KF) as predictors has a quite extensive use in industry such as sensorless control, diagnosis, and fault-tolerant control of ac drives; distributed generation and storage systems; robotics, vision, and sensor fusion techniques; applications in signal processing and instrumentation; and real-time implementation of a KF for industrial control systems [6].

The KF is extensively applied in mobile communication systems, as illustrated by the following examples. In the early stages of 4G, Aronson et al. [4] investigated the performance of various memoryless and memory-based channel estimators within different Orthogonal Frequency Division Multiple Access (OFDMA) subcarrier allocation schemes and pilot patterns, aiming to identify suitable subcarrier allocations. Furthermore, applications extended to rate adaptation in multihop networks [2] and modeling prediction uncertainties for energy-efficient stored video streaming while maintaining ideal Quality of Service (QoS) for the application [5].

A short-term predictor, designed to operate on a 10 ms scale, forecasts average throughput rates utilized by prominent schedulers [22, 23]. These simulations demonstrated elevated throughput rates and decreased packet loss ratio (PLR) with minimal impact on packet delay. However, this improvement comes at the expense of an uneven resource distribution among users.

In the context of 5G and beyond, the application of machine learning (ML) or artificial intelligence (AI) to address mobile network challenges is commonplace [29]. This includes resource allocation (RA) and traffic/throughput prediction with 95% accuracy, as demonstrated in Refs. [25, 13]. Kalman filters, either in conjunction with fuzzy logic [16] or as an auxiliary method to estimate prediction variances through optimization [5], have been utilized. However these are costly approaches, computationally speaking. And this was one of the motivators for us to make predictions and optimize network performance using a model-free predictor, the Kalman-Takens filter (KTF).

TheKTF made its debut in the work of Hamilton, Berry, and Sauer [8] as an approach to predict the results of the Lorenz '96 model [15]. Combining Takens theorem with the unscented form of the Kalman filter, the KTF functions as a model-free predictor, yielding predictions comparable to those obtained when using the Lorenz '96 equations with the unscented Kalman filter [11, 10].

The SNR prediction is very important in the mobile communication systems because it directly impacts the quality and reliability of communication. It helps to determine the expected QoS, resource allocation, handover decisions, network planning, data transmission rates, and overall user experience in wireless networks. Traditional methods often lack accuracy and adaptability in dynamic environments, as the mobile environments are dynamic with varying interference levels, user mobility, and changing channel conditions. Simple models fail to account for these variations accurately. When compared with machine learning techniques like KTF, it offers more robust solutions by learning from past data and adapting to real-time conditions effectively, which is fundamental in achieving higher QoS, efficient resource allocation, and seamless network operations in modern mobile networks.

Leveraging the KTF to predict signal-to-noise ratio values presents an opportunity to enhance the resource allocation scheme. Predicted values can guide the selection of a modulation and coding scheme (modulation and coding scheme (MCS)), allowing the system to opt for a lower MCS in environments with reduced interference or noise, based on the predicted SNR. The relationship between SNR and the selection of a modulation scheme is detailed in Refs. [17, 28]. An implementation of the KTF is presented in Ref. [21], where a simple method to predict future SNR values based on the KTF is detailed. This approach requires no additional hardware or features, as proposed in [3], facilitating a straightforward implementation in radio access networks. Furthermore, this method eliminates the need for solving optimization problems, training neural networks, or employing complex artificial intelligence algorithms for resource allocation, which may otherwise consume significant system resources and impact network performance [18].

Utilizing the KTF for forecasting SNR values can enhance the performance of a mobile network by enabling the precise selection of a MCS. This proactive approach, in contrast to the conventional reactive method, allows the system to opt for a lower MCS (i.e., reduced error correction and heightened throughput) if the predicted SNR indicates a radio environment with diminished interference and/or noise. A successful implementation of the KTF predicting SNR is exemplified in [24].

This paper is organized as follows: in Section 2, we provide a brief review of the Kalman-Takens filter; in Section 3, we present numerical results; and finally, in Section 4, we offer our concluding remarks.

## 2 Kalman-Takens model-free predictor

The KTF uses a training sequence in the interval $[1, 2, \ldots, T]$ to predict the state of the system in its next steps $[T+1, T+2, \ldots, T_f]$, where $T_f$ is the final instant. This direct prediction is performed by finding the $\kappa$-nearest neighbors of the delay coordinate vector, as given by Eq. 1:

$$\begin{aligned}
\xi_k(T^{'}) &= \left[z_k(T^{'}), z_{k-1}(T^{'}), \ldots z_{k-(d-1)}(T^{'})\right], \\
\xi_k(T^{''}) &= \left[z_k(T^{''}), z_{k-1}(T^{''}), \ldots z_{k-(d-1)}(T^{''})\right], \\
&\vdots \\
\xi_k(T^{\kappa}) &= \left[z_k(T^{\kappa}), z_{k-1}(T^{\kappa}), \ldots z_{k-(d-1)}(T^{\kappa})\right].
\end{aligned} \tag{1}$$

The superscripted prime in previous equations represents the first, second, third, etc until the $\kappa$-nearest neighbor of $\xi$ are located where they are found from the noisy training data. Once the neighbors are found, from a $\kappa$-nearest neighbor direct prediction model, the known values from the historical series are used with a local model to predict the values that would occur at future times. This is done using an average of the future points of the lagged vectors closest to the reference. The local model $\tilde{f}$ defining the system evolution for $m$ time steps ahead is given by

$$\begin{aligned}\tilde{f}(\xi_k+m) = [&w^{(')}\xi_k(T^{'}+m)+w^{('')}\xi_k(T^{''}+m)\\ &+w^{(''')}\xi_k(T^{'''}+m)+\ldots+w^{(\kappa)}\xi_k(T^{\kappa}+m)\;],\end{aligned} \tag{2}$$

where $\left\{w^{(')}, w^{('')}, \ldots, w^{(\kappa)}\right\}$ are the weights calculated for each one of the $\kappa$-nearest neighbors of the delay coordinate vector at time $T$. The weights $w^{(i)}$ are calculated as

$$w^{(i)} = \frac{e^{-\delta^{(i)}/\sigma}}{\sum_{j=1}^{\kappa} e^{-\delta^{(j)}/\sigma}}, \tag{3}$$

with $\delta^{(i)}$ being the euclidean distance from the $i$-th nearest neighbor to the delay coordinate vector $\xi$ and $\sigma = \left\langle\delta^{(i)}\right\rangle$ the average of distances $\left\{\delta^{(')}, \delta^{('')}, \delta^{(''')}\ldots\delta^{(\kappa)}\right\}$, that is, the distance of $\xi_k$ to the first, second, etc. until the $\kappa$-th neighbor [9].

Usually, we represent the state $x_k$ and the observation $z_k$ at each step $k$ of the filtering process as

$$\begin{aligned}x_k =& f(x_{k-1})+\mu_{k-1}\ ,\\ z_k =& h(x_k), +\nu_k\ ,\end{aligned} \tag{4}$$

with $f$ and $g$ being known functions and $\mu_k$ and $\nu_k$ white noise processes with covariance matrices given respectively by $Q$ and $R$ which are, a priori, unknown.

The KTF follows from the unscented Kalman filter (UKF) algorithm changing

$$f(.) \to \tilde{f}(.),$$

with $\tilde{f}$ given by Eq. (2). We calculate the sigma points $\mathcal{X}$ of the UKF as follows

$$\begin{aligned}&\mathcal{X}_{k-1}^{(0)} = \hat{x}_{k-1},\\ &\mathcal{X}_{k-1}^{(i)} = \hat{x}_{k-1}+\sqrt{P_{k-1}} \qquad i=1,\ldots,n\\ &\mathcal{X}_{k-1}^{(i)} = \hat{x}_{k-1}-\sqrt{P_{k-1}}, \qquad i=n+1,\ldots,2n\end{aligned} \tag{5}$$

where $P_k$ is the covariance matrix of the state estimation error.

Then the estimated state $\hat{x}_k^-$ and covariance $P_k^-$ are calculated as follows [10]

$$\begin{aligned}\hat{x}_k^- =& \sum_{i=0}^{2n} W^{(i)}\tilde{f}\left(\mathcal{X}_{k-1}^{(i)}\right)\ ,\\ P_k^- =& \sum_{i=0}^{2n} W^{(i)}\left[\tilde{f}\left(\mathcal{X}_{k-1}^{(i)}\right)-\hat{x}_{k-1}^-\right]\\ &\times\left[\tilde{f}\left(\mathcal{X}_{k-1}^{(i)}\right)-\hat{x}_{k-1}^-\right]^T+Q\ ,\end{aligned} \tag{6}$$

with the weights subject to $\sum_{i=0}^{2n} W^{(i)} = 1$. At each step of the filter the estimates given by Eqs.(6) are updated. Hence, we first calculate the cross covariances of state and observation, $P^{xz}$, and the covariance of observations $P^{zz}$ as

$$\begin{aligned}P_k^{xz} =& \sum_{i=0}^{2n} W^{(i)}\left[\tilde{f}\left(\mathcal{X}_{k-1}^{(i)}\right)-\hat{x}^-\right].\left[z_k-\hat{z_k}^-\right]^T\ ,\\ P_k^{zz} =& \sum_{i=0}^{2n} W^{(i)}\left(z_k-\hat{z_k}^-\right).\left(z_k-\hat{z_k}^-\right)^T+R\ ,\end{aligned} \tag{7}$$

with $\hat{z}_k^- = \sum_{i=0}^{2n} W^{(i)}z_k$. Then we calculate the Kalman gain $K_k$ and update the state estimates $x_k$ and covariances $P_k$ using

$$\begin{aligned}K_k =& P_k^{xz}/P_k^{zz}\ ,\\ \hat{x}_k =& \hat{x}_k^-+K_k\left(z_k-\hat{z}_k^-\right)\ ,\\ P_k =& P_k^- - K_k.P_k^{zz}.K_k^T\ .\end{aligned} \tag{8}$$

After the update, step $k$ is incremented, the filter goes back to Eq.(5) and repeats the preceding steps, Eqs.(6)-(8).

## 3 Numerical results

In this work we use of the KTF to forecast values of SNR. This model-free filter was introduced by Hamilton, Berry and Sauer [8], combining the filtering equations of Kalman filter[12], in particular the UKF [11, 10] with the data-driven modeling approach of Takens [20]. This innovative procedure substitutes the model with dynamics reconstructed from delay coordinates, all while utilizing the Kalman update formulation to reconcile new observations. Its was successfullly used to denoise the Lorenz'96 model [14].

A straightforward "experiment" facilitates the comprehension of how the KTF operates. In this instance, a sine function with the addition of normal noise was utilized as a time series for data generation. The variable *A* signifies the amplitude of the noise, *d* represents the number of past points used for prediction, $N_p$ denotes the quantity of points to be forecasted, and $k$ indicates the number of nearest neighbors.

A dataset from a 4G network collected by the Queen's Telecommunications Research Lab (TRL) [7] was used for the experiment. The data comes from Kingston Transit Express Bus 502 public bus route in Kingston, Ontario, Canada, and two cell phones from the same operator and Android operating system were used during the route traveled to collect it(Samsung Galaxy S9 and Samsung Galaxy S10). The measurements were recorded on three bus trips every day of the week (Monday to Friday) at 9 am, 12 pm and 6 pm for ten days. Each trip had the same route and the same starting and ending points, taking around one hour to complete and starting and ending at almost the same time every day, as shown in Fig. 2.

To work with all this information, we filtered and cleaned the data. For the tests, we selected data from one day's journey on one of the cell phone models (Samsung Galaxy S9) and extracted the information collected from SNR, downlink, and uplink. The SNR information was exported and applied as the time series to be analyzed by the Kalman-Takens Filter to predict SNR in future points in the Matlab software.

Different combinations of parameters were assigned in the KTF, with kNN representing the value of k and del the value of d. The first tests used combinations of k = 10, 20, and 30 and d = 3, 4, and 6, with a forecast value equal to 10, obtaining the results shown in Figure 1. One can observe satisfactory results, especially when optimizing the number of nearest neighbors and d combinations.

After that, different forecast values were applied, from 100 to 800 in steps of 100, which showed that when a very high forecast value is selected, the filter loses its accuracy and presents divergences in the predictions, which can be seen in Figures 3 and 4.

## 4 Final Remarks

In this work our application of the KTF for SNR prediction revealed promising outcomes, showcasing the filter's adaptability to real-world datasets. The experiment, conducted on a 4G network dataset from bus trajectory, underscored the importance of parameter tuning, especially in optimizing the number of nearest neighbors ($k$) and past points ($d$). The filter exhibited robust performance in maintaining accuracy within reasonable forecast value ranges. However, caution is advised with excessively high forecast values, as the filter's precision diminished under such conditions. Despite this limitation, the KTF emerges as a valuable tool for predicting SNR, offering practical insights for its application in telecommunications and network optimization. Note that the SNR data was considered as an example in Ref. [24], the same method may be applied to the uplink and downlink throughput data that are also collected by the mobile device during the experiment. Predicting the data rates would allow for an adaptive bandwidth allocation instead of model-based schedulers [21].

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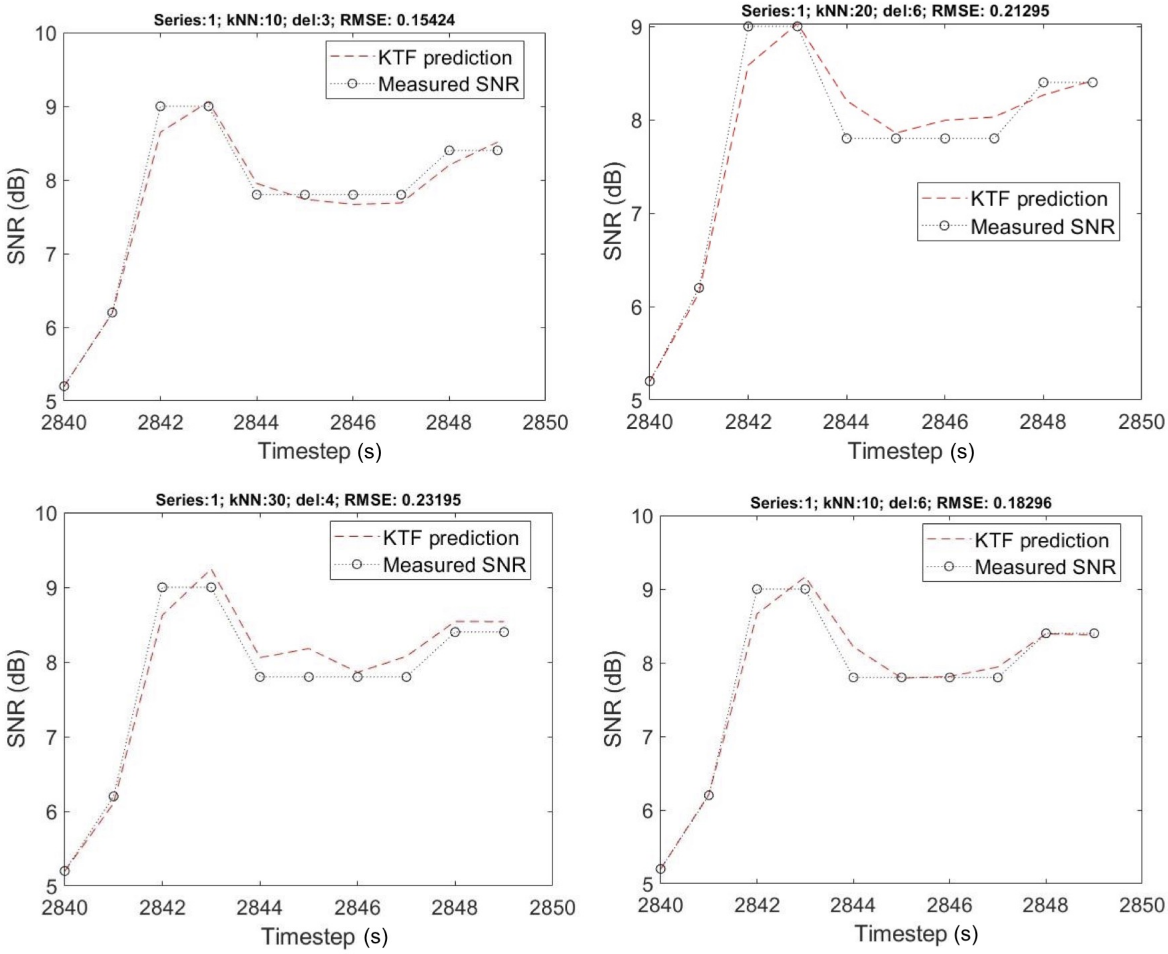


**Figure 1:** Set of tests applied to the KTF in Matlab, with combinations of values of k equal to 10, 20 and 30 and d equal to 3, 4 and 6.

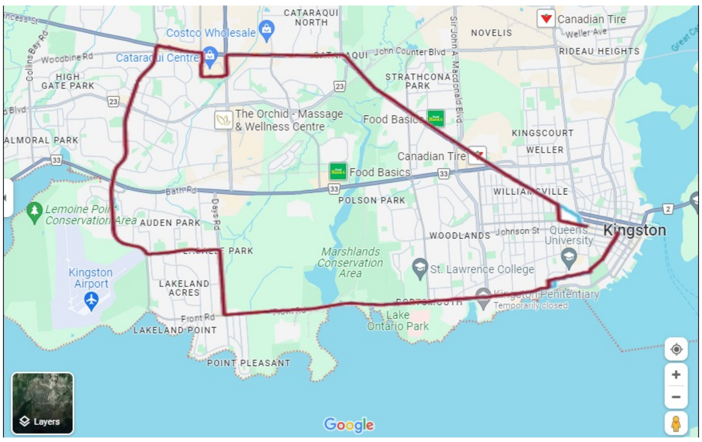


**Figure 2:** Route taken by the bus in the three measurements of a day based on latitude and longitude data in Matlab software, plotted under the city map of Ontario, Canada

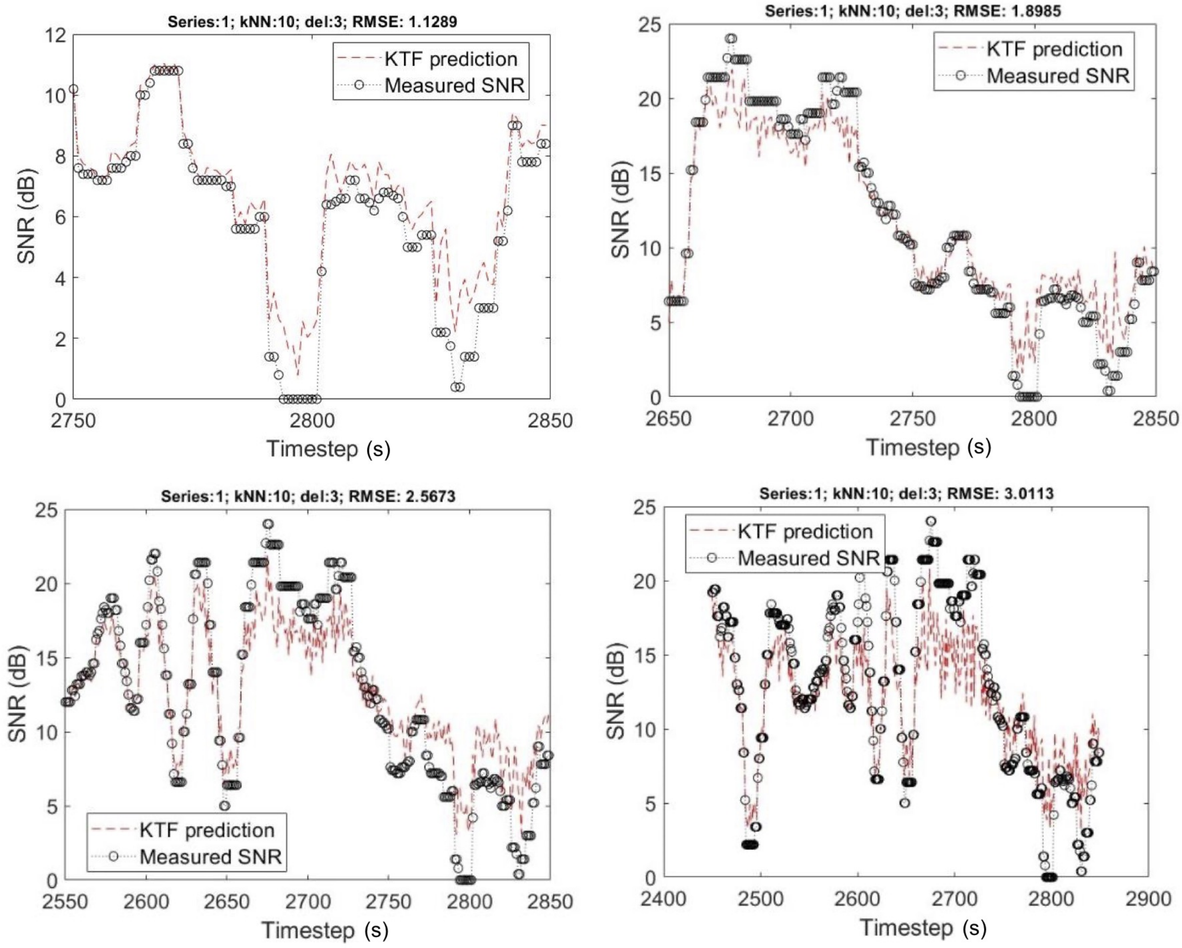


**Figure 3:** Set of tests applied to the KTF in Matlab, with fixed values for k equal to 10, d equal to 3, and the forecast variating in 100, 200, 300 and 400.

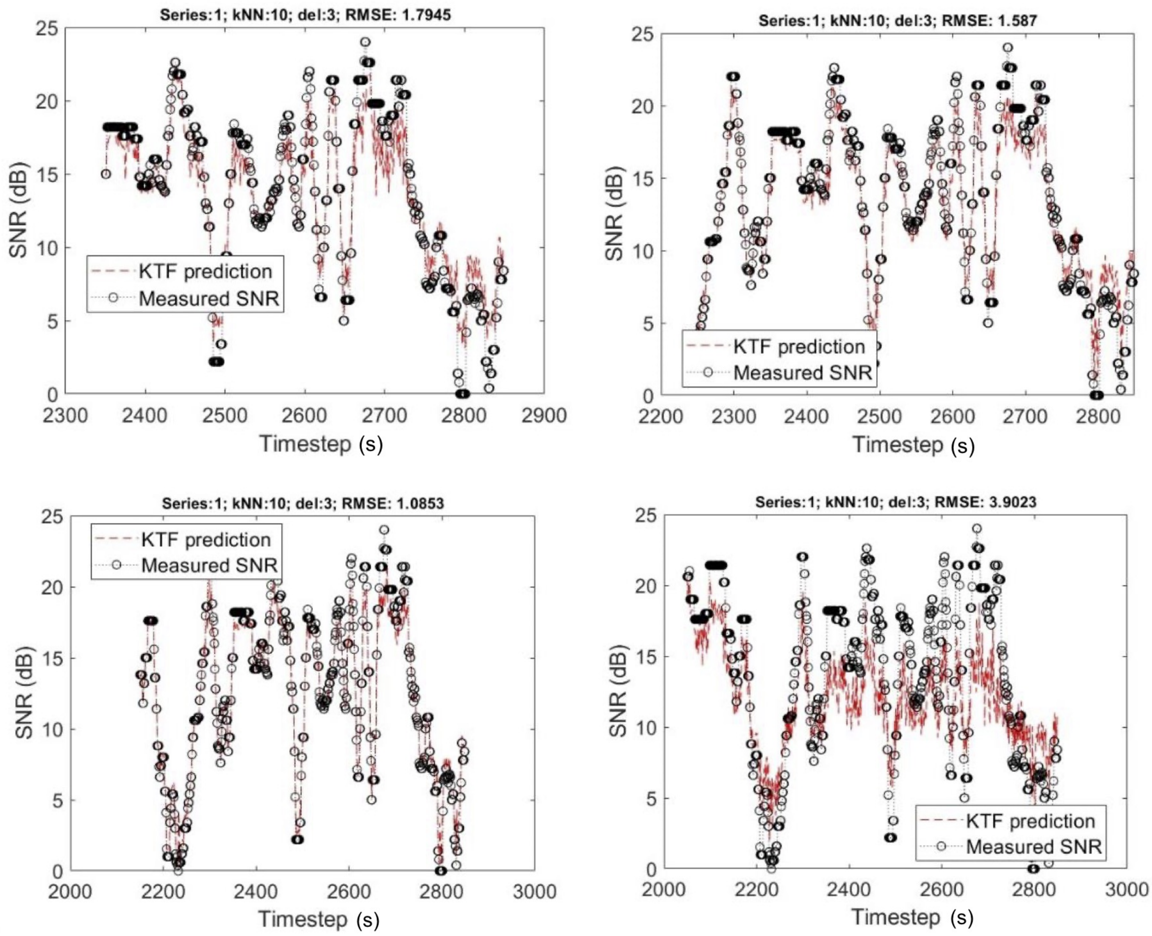


**Figure 4:** Set of tests applied to the KTF in Matlab, with fixed values for k equal to 10, d equal to 3, and the forecast variating in 500, 600, 700 and 800.