Optimal Dispatch Programming of Hydroelectric Power Generation with the use of Genetic Algorithms

CAROLINA GIL MARCELINO[1]
ELIZABETH FIALHO WANNER[1]
PAULO EDUARDO MACIEL DE ALMEIDA[1]
GRAY FARIAS MOITA[1]

[1]CEFET-MG - Centro Federal de Educação Tecnológica de Minas Gerais
PPGMMC - Programa de Pós Graduação em Modelagem Matemática e Computacional
Av. Amazonas, 7675, CEP 30.510-000, Belo Horizonte, MG, Brasil
(carolina,pema,gray)@lsi.cefetmg.br,efwanner@decom.cefetmg.br

Abstract. The population increase and the growth of buying power of home appliances cause the need of electricity power to increase every year in Brazil. Electric dispatch is defined as the attribution of operational values to each generation unit inside a power plant, given some criteria to be fulfilled. In this context, an optimal dispatch schedule for hydroelectric units in energy plants provides a greater amount of energy to be generated with less consumption of water. This paper presents an optimization solution to solve this problem for an actual plant, using Genetic Algorithms. The underlying mathematical modeling is described in details and practical validation of the proposed approach is performed through simulation experiments. In the case study, results are analysed and compared to the actual system running in a real world plant. Finally, the generality of the proposed approach is discussed and possibilities of its use to solve the same problem to other hydroelectric plants are presented.

Keywords: Electric Dispatch, Optimization, Genetic Algorithms, Simulation.

(Received June 25th, 2013 / Accepted August 30th, 2013)

1 Introduction

In Brazil, hydroelectric generation is the main source of electrical energy. The country presents an array of electrical generation predominantly renewable and hydraulic generation accounts for an amount about 81,7% of the total supply. According to current estimates from the brazilian Energy Planning Company - EPC (EPE, from the portuguese "*Empresa de Planejamento Energético*"), for a horizon of 10 years, from 2012 until 2021, the average annual growth of total electricity demand in Brazil (which includes retail consumers, free consumers and auto producers) will average 4.5% per year over the period [15]. This continuous growth is considerable and researchers are looking for ways to improve the efficiency of current production processes and to account for the forecast needs.

In general, hydroelectric plants use automated control systems to operate and to manage power generation processes [2]. Typically such systems receive power demands for specific time intervals and divide this values by the number of available generation units in a plant. The electric dispatch is defined as the assignment of values for each set of operating turbine-generator unit in the power plant, given some criteria to be met as the demand for energy to be produced, units operating limits, etc. In current systems, this demand is equally distributed among the available units [1]. However, this distribution not always represents the best operating points for the units. In other words, the equal distribution does not guarantee that each generation unit is operating on its optimal operational point, in terms of efficiency. This paper proposes a novel mathematical model to calculate hydraulic losses in hydroelectric plants, through statistics nonlinear multivariable regression techniques, and analyses the use of genetic algorithms (GA) to solve the electric dispatch problem in the short term. As a case study, this problem is solved for a large hydroelectric plant operating in Brazil.

The paper is organized as follows: Section 2 details the problem of electric dispatch and "state-of-the-art" academic research. Section 3 presents the proposed mathematical model. Section 4 details the implemented algorithms. Section 5 shows the case study, experiments and comparative analysis of algorithms and, finally, Section 6 presents the conclusions regarding the achieved performance.

2 The Problem of Electric Dispatch

For the purpose of this work, a typical power system consists of three parts: the generator center, connecting systems and consumer centers. The connecting systems can be of transmission, subtransmission and distribution types. In each of these parts, there are operating limits for the existing electrical equipment in such a way as to ensure a clear and safe generation of energy to consumer centers. To ensure power generation with minimal use of water resources is a big challenge, when one considers the operational constraints of a hydroelectric plant and the connected power system. This problem can be characterized as an optimization of the electricity production efficiency or, in other words, to generate more power with less water.

To solve this problem, it is necessary to model a hydroelectric plant during its regular operation. This mathematical modeling must include operational characteristics of the plant under study, and incorporating inherent hydroelectric penstocks losses in the model is crucial to obtain practical parameters for describing the operation of generation units with respect to water consumption and energy generation.

2.1 State of the Art

Finardi [7] proposed a mathematical model to solve the dispatch problem for hydroelectric generating units. The developed modeling uses a target amount of water being discharged by each unit of the power plant. Considering the functional non-linearity of the generation units and the presence of forbidden zones of operation, the proposed approach calculates optimal generation values for each unit. The results of that work showed that the adopted model fulfilled the desired optimization goal, making it an important reference to this work.

Dudek [6] used GA as an approach to solve the dispatch problem daily energy production. His work took into account operating costs of turning on and off the available generating units, showing that the occurrence of these interrupting events can bring financial damage to energy production. The proposed algorithm gives a stable and acceptable (near optimum) solution to the problem, but the computational cost of implementing it is high, even using parallel processing.

In his master dissertation, Araújo [2] used the mathematical model elaborated and described in [7]. With the application of computational intelligence techniques, he obtained feasible solutions to the resulting optimization problem. In pursuit of finding the best algorithm to satisfy the solution, several algorithms have been implemented. GA techniques presented the best results, efficiently achieving the desired power generation demands.

Baños [3] conducted a review of techniques that, so far, were used for optimization applied to the generation of renewable and sustainable energy. The study mentions various forms of energy production, among them, the hydroelectric one. To solve the dispatch problem, the papers cited by him used techniques as GA and Particle Swarm Optimization (PSO). The first conclusion of his survey was that the number of scientific papers that used optimization methods to solve renewable energy problems dramatically increased in the last years, but, in many cases, the computational cost is high, even when using parallel processing techniques.

Several optimization techniques to improve energy production efficiency in power systems were discussed in [15]. That study was motivated by the fact that the European Union signed the Kyoto Treaty, in May 2002, and since then, scholars come seeking to find new techniques to reduce by 20% the energy production until 2020, which is one of the goals of such agreement. Some of the described techniques are: Search Algorithms, Evolutionary Algorithms, Simulated Annealing, Tabu Search, Ant Colony Optimization, PSO, GA, Artificial Neural Networks and Evolutionary Programming. Among them, GA were recommended to minimize losses and to maximize efficiency. PSO algorithms were recommended for optimal power generation seeking.

Marcelino [12] proposed in her dissertation a new model of hydroelectric power production optimization, considering the inherent losses existing in the penstocks. To perform the calculation of efficiency for generation units, she proposed to perform a nonlinear multivariate regression in order to find the coefficients of a quadric function. Therefore, coefficients of the quadric function were obtained that, in turn, represented hydraulic parameters of operating generation units. Evolutionary Algorithms were used to maximize the productivity of an actual plant. Experiments have succeeded in demonstrating water economy during simulated generation processes.

2.2 Problem Modeling

In this section, the mathematical model proposed by Marcelino [12] to solve the problem of electric dispatch is described. The power production performed by an hydroelectric unit, in MW, is given by Eq. 1,

$$ph_{jt} = g \cdot \eta_{jt} \cdot hl_{jt} \cdot q_{jt}, \tag{1}$$

in which,

- ph_{jt} is the power generated by unit j at time t (MW);
- g is the acceleration of gravity ($9.8 \cdot 10^{-3} km/s^2$). It is presented here in this form in order to provide automatic conversion of power, from kilowatts to megawatts;
- η_{jt} is the global efficiency of unit j at time t ($\%$);
- hl_{jt} is the net water head of unit j at time t (m) and
- q_{jt} is the water flow rate of unit j at time t (m^3/s).

The hydraulic head of the reservoir, H_b, is given by subtracting the upstream level value by the downstream level value, for a given instant of time. This data is easily measured and delivered by common automation and control systems operating at a power plant. Therefore, the net water head hl_{jt} is nothing more than H_b subtracted by the total hydraulic losses. This work, unlike most current scientific studies, proposes a detailed mathematical model to calculate losses related to fluid friction in penstocks. Losses can be classified as distributed (Δ_{Hd}) and localized (Δ_{Hl}). According to [14],[16] and [18], the sum of the penstocks losses is given by Eq. 2,

$$\Delta_{Hjt} = \Delta_{Hd} + \Delta_{Hl}. \tag{2}$$

The distributed losses (Δ_{Hd}) are uniform in any part of a constant diameter pipe, regardless of the position of the pipe. So, the distributed load losses, due to fluid friction with the walls of the penstock along its entire length, can be obtained by Eq. 3,

$$\Delta_{Hd} = F \frac{L}{D} \frac{V^2}{2g}, \tag{3}$$

in which,

- F is the loss factor in the pipe;
- L is the length of pipe (m);
- D is the pipe diameter (m);
- V is the fluid velocity (m^3/s);
- g is the acceleration of gravity (here considered as (m/s^2).

The localized losses ($\triangle_{Hl}$), or load losses, which arise at specific points or parts of the pipe, are obtained by Eq. 4,

$$\Delta_{Hl} = \lambda \frac{V^2}{2g}, \tag{4}$$

in which,

- λ is the curve loss factor;
- V is the fluid velocity (m^3/s);
- g is the acceleration of gravity (m/s^2).

Using the concepts just discussed regarding losses inherent in penstocks, the following hydraulic loss calculation model was established. Considering that a penstock has divisions, which can be represented by straight sections and curves existing between them, the total loss Δ_{Hjt} can be mathematically modeled, according to Eq. 5, as

$$\Delta_{Hjt} = \sum_{s=1}^{S(n)} F \frac{L}{D} \frac{V^2}{2g} + \sum_{c=1}^{C(n)} \lambda \frac{V^2}{2g}. \tag{5}$$

As already mentioned, the parameter hl_{jt} is obtained by subtracting the hydraulic head of the reservoir H_b by the losses related to total hydraulic friction in penstocks (Δ_{Hjt}). Therefore, the net water head for each unit is given by Eq. 6,

$$hl_{jt} = H_b - \Delta_{Hjt}. \tag{6}$$

Finardi [7] has stated that the overall efficiency of a hydroelectric production unit can be computed as the product of a constant loss factor by the fluid flow rate at the turbine. However, this approach does not take into account the hydraulic losses and spill water into the turbine. So, in this work, the efficiency of a generation unit is represented by the quadratic function given by Eq. 7,

$$\eta_{jt} = \rho_{0j} + \rho_{1j}.hl_{jt} + \rho_{2j}.q_{jt}+ \quad (7)$$

$$\rho_{3j}.hl_{jt}.q_{jt} + \rho_{4j}.hl_{jt}^2 + \rho_{5j}.q_{jt}^2,$$

in which,

- η_{jt} is the global efficiency of unit j at time t (%);
- ρ_{0j}, ... , ρ_{5j} are the coefficients obtained from the Hill Diagram using multivariate nonlinear regression technique (see next section);
- hl_{jt} is the net water head of unit j at time t;
- q_{jt} is the water flow rate of unit j at time t.

2.3 Model Adjusting

The existing relationship between generated power, net water head and water flow rate is usually represented by Hill Diagrams, given by turbine manufacturers for each specific unit [7]. For the studied plant, only a unique Hill Diagram characteristic exists for all the generation units. Then, the hydraulic loss model proposed in this work becomes very relevant. It allows for the calculation of income-generating sets and to distinguish them from each other, since each machine will be uniquely characterized by a specific net hydraulic head. In order to find a good efficiency model for the plant, a nonlinear multivariable regression process was performed. A Hill Diagram of the studied plant was digitized and vectorized. Also, a computer program was built to read the resulting digital image and convert each point of the original Hill Diagram in rectangular coordinates X, Y and Z, thus generating a vector of 6,969 points (hl_{jt}, q_{jt}, η_{jt}) relating power efficiency to the inflow rates and net hydraulic heads of the generation units. As a result, Figure 1 presents the actual points generated by the Hill Diagram digitization process just described. The limits used to digitize, regarding the Hill Diagram for a Kaplan type turbine, were:

- Indep. variable: water flow rate [50, 150] (m^3/s);
- Indep. variable: net hydraulic head [32, 56] (m);
- Dependent var.: global efficiency [83, 93] (%).

With these points in hand, a specific technique of nonlinear multivariable regression was implemented. This regression process was created using the statistical toolbox of MATLAB©R2012b. This process uses the Levenberg-Marquardt algorithm [11] [13] to be performed. For the execution of this regression process, a subset of about 1,000 points arbitrarily chosen from the available set of 6,969 points, were used. Table 1 presents the coefficients obtained by the regression process, with 99% of accuracy.

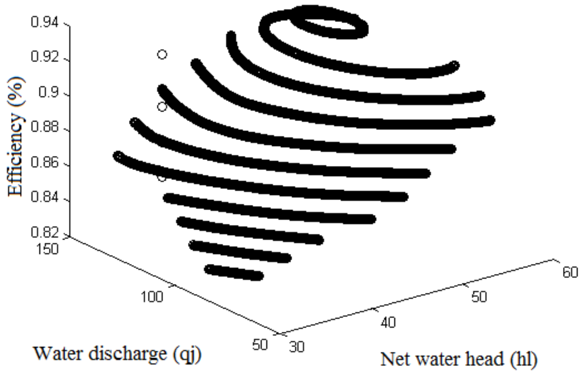

Figure 1: Hill Diagram resulted from digitization process

Table 1: Efficiency Coefficients obtained by the Regression Process

Coefficient	Value
ρ_{0j}	1.4630e-01
ρ_{1j}	1.8076e-02
ρ_{2j}	5.0502e-03
ρ_{3j}	-3.5254e-05
ρ_{4j}	-1.1234e-03
ρ_{5j}	-1.4507e-05

The validation of the coefficients shown in Table 1 and the resulting efficiency model represented by Eq. 7 can be seen in Figure 2, which shows the overlap of original (digitized) points and calculated (obtained through regression) points.

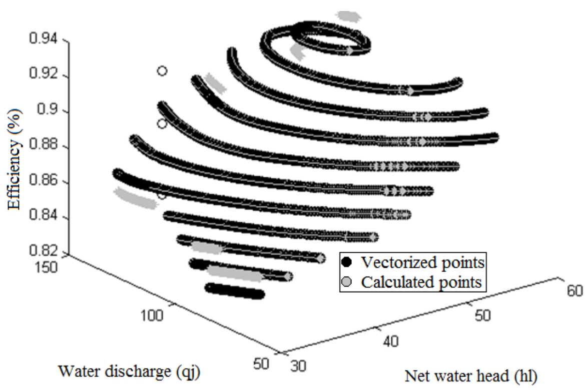

Figure 2: Original versus calculated points of the Hill Diagram

In order to test the generalization property of the proposed model, a 3D continuous curve was built, using the same parameters of Table 1. Figure 3 presents the overlap between digitized points and calculated points for the whole operational ranges of the independent variables water flow rate and net hydraulic head. The graph shows that the implemented regression process was satisfactory, as the coefficients estimated by nonlinear multivariable regression provided a Hill Diagram very similar to the original one.

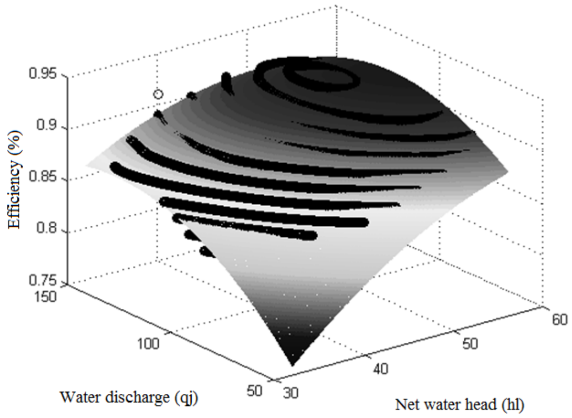

Figure 3: Generalization test for the adjusted model of the Hill Diagram

As shown by Eq. 7, power production function depends directly on q_{jt} and hl_{jt}. But as the net hydraulic head hl_{jt} can be approximated by a function of the water flow rate q_{jt}, Finardi [7] proposed a simplified production function as a polynomial of seventh degree in terms of coefficients associated with q_{jt}, according to Eq. 8. This function was also used in the model proposed by Araújo [2].

$$p_{jt}(q_{jt}) = \rho_{0j} q_{jt} + \rho_{1j} q_{jt}^2 + ... + \rho_{6j} q_{jt}^7. \tag{8}$$

According to Finardi [7], coefficients ρ_{0j} ,..., ρ_{6j} are parameters dependent on operating characteristics and are calculated using the Hill Diagram, losses in penstocks and gross losses, among others. But using the efficiency model proposed by Finardi, only 6 operating coefficients can be obtained, and his function uses 7 coefficients. Then, it becomes impossible to solve the problem, making use of this function, without further information about system conditions. Another relevant fact is that the author does not explain or justify the construction of his model. Given this modeling problem, this paper proposes a different function for calculating electric power, according to Eq. 9,

$$ph_{jt} = g \cdot [\rho_{0j} + \rho_{1j} hl_{jt} + \rho_{2j} q_{jt} + \rho_{3j} hl_{jt} q_{jt} + \rho_{4j} hl_{jt}^2 + \rho_{5j} q_{jt}^2] \cdot [Hb_{jt} - \Delta_{Hjt}] \cdot q_{jt}, \tag{9}$$

in which,

- ph_{jt} is the power generated by unit j at time t;
- g is the acceleration of gravity ($9.8 \cdot 10^{-3} kg/m^2 s^2$);
- ρ_{0j}, ... , ρ_{5j} are the coefficients obtained from the Hill Diagram using nonlinear multivariable regression technique;
- hl_{jt} is the net hydraulic head of unit j at time t;
- q_{jt} is the water flow rate of unit j at time t;
- Δ_{Hjt} are the total losses referring to the penstock connected to generation unit j at time t.

2.4 Optimization Model

According to the whole mathematical model presented so far, the goal of optimization is to maximize the hydroelectric production function, taking into account all the generating units as represented by the objective function shown by Eq. 10. The vector optimization variables are represented by the water flow rate of each generation unit,

$$x = [q_{1t}; q_{2t} ... q_{jt}].$$

$$Maximize\ F(x) = \frac{\sum_{j=1}^{J(r)} ph_{jt}}{\sum_{j=1}^{J(r)} q_{jt}}, \tag{10}$$

subject to:

$$\sum_{j=1}^{J(r)} ph_{jt} \cong D,$$

$$q_{jt}min \leq q_{jt} \leq q_{jt}max,$$

$$ph_{jk}^{min} \sum_{k=1}^{\emptyset_j} Z_{jk} \leq ph_{jt} \leq ph_{jk}^{max} \sum_{k=1}^{\emptyset_j} Z_{jk},$$

$$Z_{jk} \in \{0, 1\}, \sum_{k=1}^{\emptyset_j} Z_{jk} \leq 1.$$

This objective function determines how much power the plant is able to produce with a given volume of water. To maximize this function means to produce more power using less water. The numerator is the production function: as this value grows, the objective function increases its value. The denominator, when reduced, also increases the value of the productivity ratio. This fractional is subject to operative demand constraints, i.e., the sum of production from all generation units must be equal to the total power demand required to be produced by the plant. The power production must also comply with the operational limits of generation units, represented by the inequality constraints of the objective function.

The first constraint indicates that the power to be delivered should be equal to the power requested by consumer electric demands; the National System Operator (ONS, from the Portuguese name “Operador Nacional do Sistema”), a government agency which operates the whole power production of Brazil, accepts an error rate of up to 0.5% above or below the demanded power. The second constraint states that the calculated flow rate must comply with the minimum and maximum capacity of each generation unit. The third constraint requires that the corresponding generated power complies with the minimum and maximum capacity of each generation unit. At last, the fourth constraint insures that each generation unit maintains its operating zone, i.e., stays on or off during the whole production period.

To optimize the proposed objective function, this work proposes the use of GA, a sub-field of evolutionary computation proposed by James Holland [10] in the seventies, in order to find optimal water flow rates for each generation unit and seeking to decrease the value of the optimization variables as low as possible, while maintaining the desired constraints still valid.

3 Optimization Algorithms

An algorithm is a sequence of executable actions to obtain a solution to a particular problem. In the context of Operational Research, algorithms are practical implementations of optimization methods, whose goal is to determine the solutions for a specific problem. This paper proposes the use of GA to solve the discussed optimization problem (see Eq. 10).

3.1 Genetic Algorithms

GA are a metaheuristic technique used in computer science to find approximated solutions to optimization and search problems. GA are a particular class of evolutionary algorithms that use operations inspired by evolutionary biology such as inheritance, mutation, natural selection and crossover [9]. GA are implemented as a computer program in which a population of abstract representations of solutions to a given problem is evolved in a search of better solutions. The evolution usually starts from a set of randomly created solutions and is carried through generations. At each generation, the adjustment of each individual (or solution) in the population is evaluated. Some individuals are selected for the next generation, and mutated or recombined to form a new population.

The new population is used as input for the next iteration of the algorithm. This loop is executed until candidate solutions meet the expected outcome by the implemented fitness function. The binary representation is the basic way to translate the actual problem in a viable way to be processed by the computer program. Importantly, the representation of a chromosome (or individual) is critical to GA. The works [2], [3] and [15] reported that the use of GA gave satisfactory results for the problem of electric dispatch.

Considering those reports, this technique is applied as the main strategy to find good solutions for the problem. Note that this problem is not simple to solve with conventional techniques. These algorithms are based on unrealistic assumptions of linearity and convexity, which cannot be assumed in the case of nonlinear problems. This work implements two versions of GA: the first one uses binary representation of solutions as individuals (called BGA) and other uses real representation of solutions as individuals (called RGA). Figure 4 ilustrates the difference between both approaches.

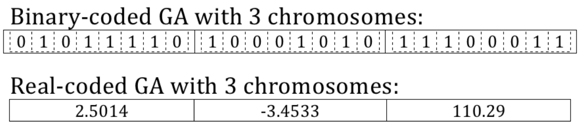

Figure 4: Binary and real-valued approaches of GA

To solve the dispatch problem, GA algorithms create populations of water flow rates, where each individual represents a feasible water flow rate for each generation unit in the power plant. The adopted stop criterion is the number of iterations. The “Canonical GA”, as proposed by Goldberg [8], was chosen to be implemented in both cases. The adopted fitness function is the objective function (see Eq. 10) with an additional penalty equal to 0.5. Details of each implemented version are depicted in the next sections.

3.1.1 BGA - Operators and Parameters

The crossover operator implemented in BGA uses single point crossover (a crossover point is chosen, the binary string from the beginning of the chromosome to the crossover point is copied from the first parent and the rest copied from the other parent). The mutation operator uses inversion bit (some bits of the chromosomes are reversed). Individuals have a length of 16 bits. BGA uses the parameters presented by Table 2.

Table 2: Parameters used by GA

Parameter	Value
Population size	50
Crossover probability	60%
Mutation probability	2%
Exchange bit probability	50%
Gamma adjustment function	1.8
Maximum number of generations	50

3.1.2 RGA - Operators and Parameters

The crossover operator implemented in RGA uses the Simulated Binary Crossover (SBX) algorithm, as proposed by [5]. SBX is designed respecting the properties of single crossing point, but by averaging the values of the individuals, it estimates the best cut-off point for each crossing. The mutation operator uses a function polynomial which defines the best gene to be mutated, as proposed also by [5]. Individuals are represented by 1x6 real valued vectors. RGA used the same parameters as the BGA algorithm (see Table 2).

4 Case Study

The case study of this paper is a hydroelectric plant installed in Brazil, with a nominal power production capacity of about $400MW$. This section will discuss the general characteristics of this plant, which operates with 6 generation units, as well as the experiments performed to solve the electric unit dispatch problem, with the use of two GA methods, as discussed in the preceding sections.

To simulate the plant behaviour, the efficiency model used the same coefficients shown in Table 1. The inputs to the algorithm are an hourly power demand generation order to be delivered by the plant and the hydraulic head of the reservoir H_b, at the time of generation. The value of H_b for the plant in question ranges between 32 and 56m. All generation units are considered as identical, so the Hill Diagram coefficients are the same. The facility has other constraints as water flow rates per unit q_{jt} and generated power per unit ph_{jt}, namely:

- q_{jt} must be in the range [70, 140] m^3/s;
- ph_{jt} must be in the range [35, 66] MW.

4.1 Practical Experiments

To validate the model proposed in this work, this section presents two experiments performed with parameters above described. As a first experiment, a test of daily demand was executed to verify the behaviour of GA algorithms while trying to meet the demand and to minimize generation units' water flow rates. The second experiment tested the hourly demand situation, which checked evolution behaviour of the proposed algorithms while they were trying to meet the demand saving water discharge. After all, an statistical analysis was performed to objectively verify what is the best approach to solve the problem.

The algorithms here described were implemented using MATLAB©R2012b. The experiments were performed on a Intel Dual Core 2.1GHz processor machine, with 3GB of RAM, running MS-Windows.

4.1.1 Experiment 1: Daily Demand

To demonstrate the feasibility of the solutions obtained by GA algorithms, when implemented as discussed in Section 3.1, an example of behaviour that shows a random daily demand follows. Simulations using all GA strategies have quite the same performance in terms of meeting the existing power constraints, as shown by figures 5 and 6. This experiment used a hydraulic head of the reservoir H_b of $54m$.

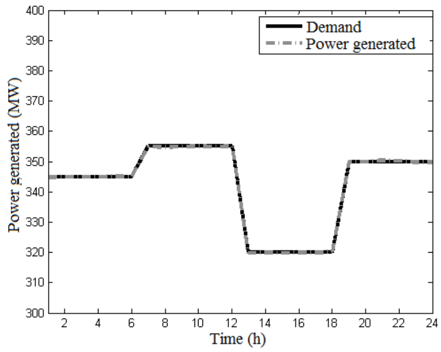

Figure 5: Typical plot of power generated through time

As can be seen in Figure 5, the generated power demand and required power demand curves are overlapped, certifying that the algorithm fulfilled the power production requirements. Moreover, it is clear in this experiment that the the total water discharge provided by the algorithm solution (identified by "determined by the algorithm" in the figure) was lower than the conventional solution (identified by "control mode" of operation in the figure), which corresponds to power demand equally distributed among the available generation units. As shown by Figure 6, it is noticeable that the proposed "optimized mode" saves water during generation, compared to the conventional "control mode".

To check the behaviour of each generation unit, per unit graphics were generated and presented in figures 7

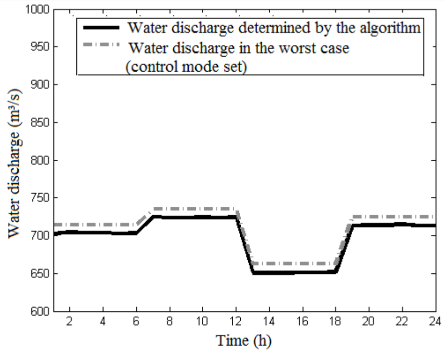

Figure 6: Typical plot of water flow rate through time

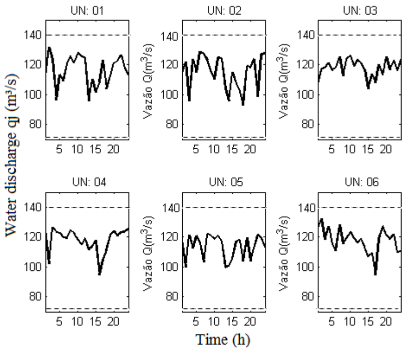

Figure 8: Typical plot of water flow rate through time, per unit

and 8. As can be seen, the algorithm respects the limits of power and water flow rates set by the system constraints, because the values found for both, as for power as for water flow rates, are between the dotted lines, which represent in the graphics the respective limits of these variables.

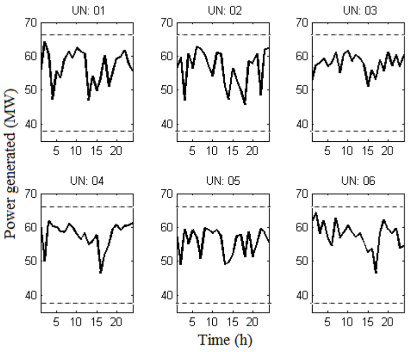

Figure 7: Typical plot of power generated through time, per unit

This result shows that it is possible to optimize hydroelectric operation applying different power demands for each generation unit inside a plant. It then contributes to the deconstruction of the hypothesis raised by [17], that "the optimal operating point of a hydroelectric plant is achieved only when the generation demand is equally divided by the number of generation units".

4.1.2 Experiment 2: Hourly Demand

The main goal of this experiment is to find the average processing time of each algorithm to achieve an optimized solution to dispatch problem. To check the behaviour of BGA and RGA algorithms, a demand of $320MW$ was established, since this is a typical demand of the plant. The hydraulic head of the reservoir, H_b, was set to $54m$. Figures 9 and 10 show plots of typical behaviour of the fitness functions for each algorithm, throughout generations, while they tried to maximize the plant productivity.

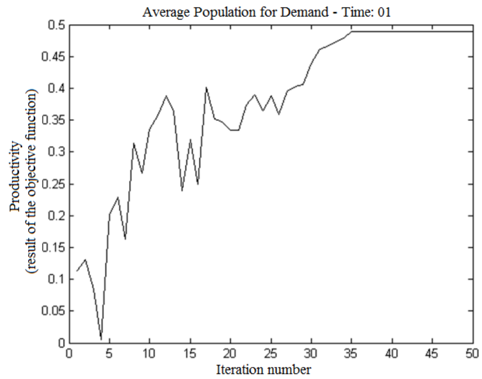

Figure 9: BGA - Evolution of fitness function through generations

It is clear from the figures that both algorithms converged to the best result since the 35th generation. One may notice that there is more sinuosity in BGA, which features higher falls than RGA. This means that for BGA it is harder to achieve stability. RGA presents lower sinuosity when compared to BGA. This fact indicates that RGA algorithm was able to obtain more confident results at each iteration, until reaching convergence at the 35^a generation.

A typical simulation table report for BGA and RGA results is shown by Table 3. It presents the results for the best individual obtained by water flow rate q_{jt} and, through these values, other parameters are calculated from the mathematical model.

In this experiment, the demand equally divided by the number of units is $53.33MW$ per unit, which corresponds to an unitary water flow of $109.175m^3/s$. In this context, the productivity of plant "control mode" for this experiment is 0.48. The values found by GA algorithms after maximizing productivity for this experiment were 0.4904 (by BGA) and 0.4905 (by RGA). So, RGA algorithm achieved the highest savings rate

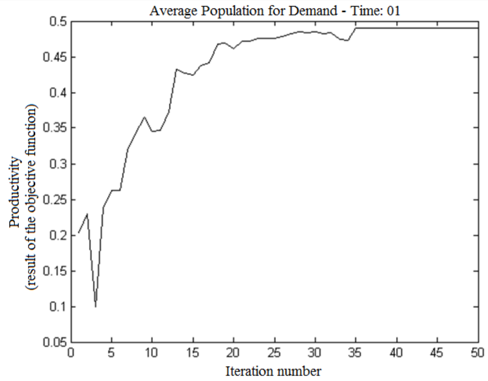

Figure 10: RGA - Evolution of fitness function through generations

Table 3: General Experiment Report

BGA | $H_b = 54m$					
UN	$ph\,(MW)$	$q\,(m^3/s)$	$\eta\,(\%)$	$hl(m)$	$\Delta_H\,(m)$
1	49,719	101,41	**0,93**	53,796	0,20377
2	59,383	121,04	**0,93**	53,83	0,17007
3	62,218	126,81	**0,93**	53,833	0,16745
4	44,282	90,253	**0,93**	53,834	0,1656
5	57,078	116,33	**0,93**	53,834	0,1656
6	47,512	96,838	**0,93**	53,833	0,16745
SUM	**320,19**	**652,68**	Demand request: 320 (MW)		
SUB	**0,19**	**2,37**	Mode S: 655,05 (m^3/s)		
RGA | $H_b = 54m$					
UN	$ph\,(MW)$	$q\,(m^3/s)$	$\eta\,(\%)$	$hl(m)$	$\Delta_H\,(m)$
1	51,047	104,14	**0,93**	53,785	0,21489
2	47,343	96,515	**0,93**	53,821	0,17935
3	58,589	119,44	**0,93**	53,823	0,17659
4	49,178	100,25	**0,93**	53,825	0,17464
5	60,31	122,94	**0,93**	53,825	0,17464
6	53,653	109,37	**0,93**	53,823	0,17659
SUM	**320,12**	**652,65**	Demand request: 320 (MW)		
SUB	**0,12**	**2,4**	Mode SC: 655,05 (m^3/s)		

and consequently the highest rate of productivity, corresponding to a water flow rate of $2.4m^3/s$. Expanding to one hour, this is equivalent to approximately 8.6 million litres of water. It is also easy to check that, in "optimized mode" of operation, all units reached maximum efficiency of 93% with use of water flow rate determined by the algorithms.

4.2 Comparative Analysis of GA Algorithms

In order to ensure the central limit theorem of normality, each execution of the algorithms was repeated for thirty times. Carrano [4] showed that evolutionary algorithms cannot be compared only by means of computational performance. Being stochastic search heuristics, it is feasible that each execution have a different result. With this in mind, a comprehensive analysis of the results provided by GA approaches were developed and tested, by means of statistical inference and multiobjective tools, as discussed in the following sections.

4.2.1 Tukey Test

To perform an objective analysis of the multiple obtained results sets, this study used analysis of variance (ANOVA) by means of Tukey Test, to find relevant information that could differentiate the tested algorithms. This statistical method can be interpreted as a comparison between means of different groups of solutions and the variances of all individuals within those groups. Tukey's strategy is to define the least significant difference between these means. The hypothesis to be considered in this test is the equality of results of the series of datasets provided by BGA and RGA algorithms, adopting a confidence interval of 95%. Table 4 shows the results of the performed Tukey Test. It indicates that the hypothesis of equality between means of factors it not rejected, because P-value is close to zero. In other words, this test indicated that the results of the compared algorithms did not have sufficient statistical evidence to be considered as different (better or worse) from each other.

Table 4: Tukey Test: BGA x RGA

Levels	**Center**	**Min**	**Max**	**P-value**
BGA-RGA	0.00053	0.00025	0.00816	0.00029

4.2.2 Multiobjective Analysis of a Mono-objective Problem

So far, this work approached the electric dispatch problem in a mono-objective way. This section proposes an multiobjective (MO) analysis for this mono-objective problem, considering the value of the objective function and the computational time of each one of the algorithms as two new objectives to be simultaneously used while comparing them. Here, again, the executions of each algorithm were repeated for thirty times, to achieve statistical validation of the experiments. To differentiate solutions obtained in an MO analysis, an approach quite widespread in the literature is the concept of "Pareto Dominance".

According to Carrano [4], the concept of Pareto Dominance can be used to compare feasible solutions to a problem. Given two solutions, x and y, it is said that x dominates y (denoted $x \succeq y$) if the following conditions are met:

1. The solution x is at least equal to y for all purposes;
2. The solution x is greater than y for at least one goal.

Thus, there is a set of solutions that have advantage over others, an optimal set of alternatives that are non-dominated by each other. The set of non-dominated solutions to a problem is usually called in the literature by a "Pareto Front". For this analysis, a graphic was generated containing the Pareto Fronts obtained by both algorithms, as it is shown in Figure 11.

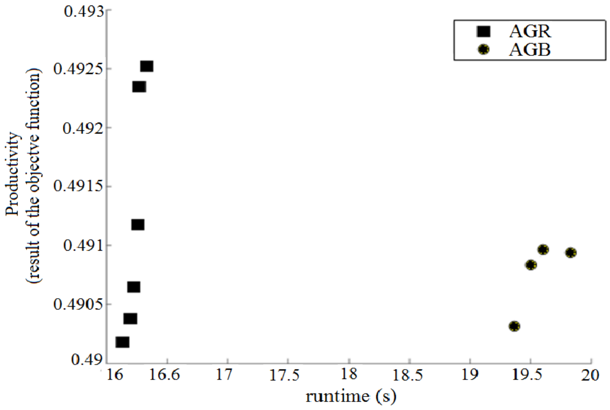

Figure 11: Comparative Pareto Front: BGA x RGA

It is noticeable from the graphic that the algorithm with the best Pareto Front is RGA, because its set of solutions has the lowest execution time, around $16.0s$, and yet produced increasing values of power productivity above 0.4925. It is also clear that the Pareto Front of RGA algorithm dominates the Pareto Front of BGA algorithm. From the MO point of view, considering average execution time as a second goal,it can be stated that RGA performed better than BGA algorithm, while solving the electric dispatch problem in this experiment.

5 Conclusion

This paper proposed a novel mathematical modeling approach to calculate hydraulic losses in penstocks at hydroelectric plants, allowing for the calculation of electrical power production in a way distinct from previous works. This formulation was used inside an optimization schema to find optimal operating points for generation units in an actual plant in Brazil. Given the discussed experimental results, it can be seen that optimization methods based on Genetic Algorithms converged to satisfactory results.

Statistical analysis indicated that the approach using real representation of individuals (called RGA) showed better results than the approach using binary individuals (called BGA) to solve the problem of electric dispatch. The productivity indexes found in "optimized mode", using BGA and RGA, are higher than the value of the same index when running the plant in conventional "control mode". This finding assures the relevance of the approach adopted in this work. Finally, the simplicity of the proposed model and the small amount of operational data necessary to implement it in practice indicate that this approach can easily be applied to other case studies, what makes it fairly general.

Acknowledgement

The authors would like to thank CEMIG and Orteng, for the cession of operational data and support, ANEEL and CEMIG for the grant GT333-2011, and CEFET-MG for the infrastructure used in this project.

References

[1] Almeida, P. E. M. Optimization of voltage and power ensemble control for electric energy generation with use of artificial intelligence. Technical report, Annual Research & Development Program CEMIG/ANEEL - Cycle 2007/2008, 24p (in Portuguese), 2007.

[2] Araújo, R. Hydroelectric energy production modeling and optimization: an approach with the use of intelligent systems. Master's thesis, CEFET-MG (in Portuguese), 2010.

[3] Baños, R. Optimization methods applied to renewable and sustainable energy: a review. *ELSEVIER, In: Renewable and Sustainable Energy Reviews*, 15:1753–1766, 2011.

[4] Carrano, E., Wanner, E., and Takahashi, R. A multicriteria statistical based comparison methodology for evaluating evolutionary algorithms. *IEEE Transactions on Evolutionary Computation*, 15:848–870, 2010.

[5] Deb, K. and Agrawal, R. Simulated binary crossover for continuos search space. *Convenor, Technical Reports, 34p*, 1994.

[6] Dudek, G. Unit commitment by genetic algorithm with specialized search operators. *ESELVIER In: Eletric Power Systems Research*, 74:299–308, 2004.

[7] Finardi, E. Unit commitment of single hydroelectric plant. *Electric Power System Research*, 75:116–123, 2005.

[8] Goldberg, D. E. *Genetic Algorithms in Search, Optimization and Machine Learning*. Addison-Wesley, ISBN 978-0201157673, 1989.

[9] Goldberg, D. E., Harik, G., and Lobo, F. The compact genetic algorithm. *IEEE Trans. on Evolutionary Computation*, 3(4):287–297, 1999.

[10] Holland, J. H. *Adaptation in natural and artificial systems*. Michigan Press, ISBN 978-0262581110, 1975.

[11] Levenberg, K. A method for the solution of certain non-linear problems in least squares. *Quarterly of Applied Mathematics 2*, pages 164–168, 1944.

[12] Marcelino., C. Optimization of a ensemble control system for hydroelectric energy generation with use of computational intelligence. Master's thesis, CEFET-MG (in Portuguese), 2012.

[13] Marquardt, D. An algorithm for least-squares estimation of nonlinear parameters. *SIAM Journal on Applied Mathematics*, 11(2):431–441, 1963.

[14] Netto, A. *Hydraulics Manual*. 8a Ed, Edgard Blucher, ISBN 978-8521202776 (in Portuguese), 1998.

[15] Pezzini, P. Optimization techniques to improve energy efficiency in power systems. *ELSEVIER In: Renewable and Sustainable Energy Reviews*, 15:2028–2041, 2011.

[16] Porto, R. *Basic Hydraulics*. EDUSP, São Paulo, ISBN 978-8576560845 (in Portuguese), 2004.

[17] Ribas, F. Optimization of energy generation in hydroelectric plants. In *Tracbel 3º SEPOCH, Paraná (in Portuguese)*, 2002.

[18] Schreiber, G. *Hydorelectric Plants*. 2a. Ed., ENGEVIX / Edgard Blucher Ltda, Rio de Janeiro (in Portuguese), 1981.