Hybrid Particle Swarm Optimiser using multi-neighborhood topologies

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Abstract. Hybrid Particle Swarm Optimization (PSO) algorithm that combines the idea of global best model with the idea of local best model is presented in this paper. The hybrid PSO mixes the use of the traditional velocity and position update rules of star, ring and Von Neumann topologies all together. The objective of building PSO on multi-models is that, to find a better solution without trapping in local minimums models, and to achieve faster convergence rate.

This paper describes how the hybrid model will get the benefit of the strength of gbest and lbest models. It investigates when it would be better for the particle to update its velocity using star or ring or Von Neumann topologies. The performance of proposed method is compared to other standard models of PSO using variant set of benchmark functions to investigate the improvement.

Keywords: PSO; hybrid PSO; global best; local best; Neighborhood Topologies; Star, Ring, Von Neumann

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1 Introduction

Particle swarm optimization (PSO) is a population based stochastic optimization technique developed by Dr. Eberhart and Dr. Kennedy in 1995, inspired by social behaviour of bird flocking or fish schooling[7],[8],[10]. In a PSO system, a swarm of individuals (called particles) fly through the search space. Each particle represents a candidate solution to the optimization problem. The position of a particle is influenced by the best position visited by itself (i.e. its own experience) and the position of the best particle in its neighborhood (i.e. the experience of neighboring particles).

When the neighborhood of a particle is the entire swarm, the best position in the neighborhood is referred to as the global best particle, and the resulting algorithm is referred to as the gbest PSO. When smaller neighborhoods are used, the algorithm is generally referred to as the lbest PSO [12]. The performance of each particle (i.e. how close the particle is to the global optimum) is measured using a fitness function that varies depending on the optimization problem.

The effect of neighborhoods on PSO has been investigated by Kennedy [6][9]. Two common neighborhood topologies are star (or wheel) and ring (or circle) topologies. For the star topology one particle is selected as a hub, which is connected to all other particles in the swarm. For the ring topology, particles are arranged in a ring. Each particle has some number of particles to its right and left as its neighborhood. However, Kennedy and Mendes [9] proposed another PSO model using a Von Neumann topology. For the Von Neumann topology, each particle is connected to its four neighbor particles (above, below, right and left particles). Figure1 illustrates the different neighborhood topologies. The results show that the performance of the PSO can be improved using different neighbourhood topologies especially when applied to multimodal functions.

The purpose of this paper is to investigate the performance of PSO when multiple methods of nieghborhoods combined together in the same search algorithm. For each iteration, particle will update its velocity (and then its position) using the neighbourhood topology that gives better position (fitness).

The rest of the paper is organized as follows: Section 2 provides an overview for standard PSO. An overview of neighbourhood topologies is shown in section 3. The proposed algorithm is presented in section 4. Benchmark functions to measure the performance of hybrid algorithm with other strategise are provided in Section 5. The Results of the experiments are presented in Section 6. Finally, section 7 concludes the paper and provides guidelines for future research.

2 Standard Particle swarm optimization

PSO was introduced by Kennedy and Eberhart in 1995. It was inspired by the swarming behaviour as is displayed by a flock of birds, a school of fish, or even human social behaviour being influenced by other individuals [7].

PSO consists of a swarm of particles moving in an n dimensional. Every particle has a position vector ($present[]$) encoding a candidate solution to the problem and a velocity vector ($v[]$). Moreover, each particle contains a small memory that stores its own best position seen so far ($pbest[]$) and a global best position ($gbest[]$) obtained through communication with its neighbor particles.

PSO is easy to implement and has been successfully applied to solve a wide range of optimization problems such as continuous nonlinear and discrete optimization problems [7][8][12]. The swarm in PSO is initialized with a group of random particles (solutions) and then searches for optima by updating generations. In each iteration, each particle is updated by following two "best" values. The first one is the best solution (fitness) it has achieved so far. This value is called $pbest[]$. Another "best" value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the population. This best value is a global best and called $gbest[]$. When a particle takes part of the population as its topological neighbors, the best value is a local best and is called $lbest$. [15]

After finding the two best values, the particle updates its velocity and positions with following equation (1) and (2).

$$v[] = wv[] + c1 \times rand() \times (pbest[] - present[]) + c2 \times rand() \times (gbest[] - present[]) \quad (1)$$

$$present[] = present[] + v[] \quad (2)$$

where, w is the inertia weight, which was first introduced by Shi and Eberhar[12][13], $v[]$ is the particle velocity, $present[]$ is the current particle (solution). $pbest[]$ is ,as stated before, the best position seen so far by particle. Where $gbest[]$ is the global best position obtained so far by any particle. $rand()$ is a random number between (0,1). c1, c2 are acceleration constants [14], usually $c1 = c2$.

In equation (1), the first part represents the inertia of pervious velocity; the second part is the "cognition" part, which represents the private thinking by itself; the third part is the "social" part, which represents the cooperation among the particles [5]. Velocity updates can also be clamped through a user defined maximum velocity, Vmax, which would prevent them from exploding, thereby causing premature convergence [1].

The pseudo code for PSO is summarized as follows [16]:

(1) Initialize a population of particles with random positions and velocities of d dimensions in the problem space.

(2) For each particle, evaluate the fitness according to the given fitness function in d variables.

(3) Compare current particle's fitness with its previous fitness. If current value is better than the previous, then set $pbest[]$ value equal to the current value, and the $pbest[]$ location equal to the current location in d dimensional space.

(4) Compare fitness evaluation with the population's overall previous best position. If the current value is better than $gbest[]$, then reset $gbest[]$ to the current particle's array index and value.

(5) Change the velocity and position of the particle according to equation (1) and (2), respectively.

(6) Repeat steps (2) to (6) until a criterion is met, usually a sufficiently good fitness or a maximum number of iterations/epochs.

3 Neighbourhood topologies

Various types of neighbourhood topologies are investigated and presented in literature [6]. The neighbourhood topologies which are considered in this paper are:

a. Star (or wheel) topology.

b. Ring (or circle) topology.

c. Von Neumann (or Square) topology.

3.1 Star Topology

Star Topology, which is also known as gbest, is a fully connected neighborhood relation. In star topology, one particle is selected as a hub, which is connected to all other particles in the swarm. However, all the other particles are only connected to the hub. Using the gbest model the propagation is very fast (i.e. all the particles in the swarm will be affected by the best solution found in iteration t, immediately in iteration t+1). This fast propagation may result in the premature convergence problem. This occurs when some poor individuals attract the population- due to a local optimum or bad initialization - preventing further exploration of the search space [2]. Figure 1(a) illustrates the star neighborhood topologies.

3.2 Ring Topology

A Ring topology, which is also known as lbest, connects each particle to its K immediate neighbors (e.g. K = 2 (left and right particles). The "flow of information" in ring topology is heavily reduced compared to the star topology. It will for instance take 2swarmsize time steps for a new global best position to propagate to the other side of the ring. However, using the ring topology will slow down the convergence rate because the best solution found has to propagate through several neighborhoods before affecting all particles in the swarm. This slow propagation will enable the particles to explore more areas in the search space and thus decreases the chance of premature convergence. In lbest model, the particle replace the equation (1), by the following:

$$v[] = wv[] + c1 \times rand() \times (pbest[] - present[]) \\ +c2 \times rand() \times (lbest[] - present[]) \quad (3)$$

Where $lbest[]$ represent the best position found so far by K immediate particle's neighbors. Figure1 (b) illustrates the ring neighborhood topologies.

3.3 Von Neumann topology

Von Neumann topology was proposed by Kennedy and Mendes [9]. Von Neumann is also a type of lbest model. However, in Von Neumann topology, particles are connected using a grid network (2-dimensional lattice) where each particle is connected to its four neighbor particles (above, below, right, and left particles)[10]. In Von Neumann topology, particles update their velocity using equation (3). However, not like Ring topology, $lbest[]$ here represent the best value obtained so far by any particle of the neighbors (above, below, right, and left particles). Like Ring topology, using Von Newmann topology will slow down the convergence rate. Slow propagation will enable the particles to explore more areas in the search space and thus decreases the chance of premature convergence. Figure1 (c) illustrates the Von Neumann neighborhood topologies.

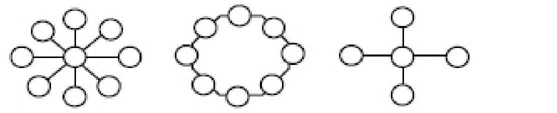

Figure 1: Representation diagram for neighborhood topologies

4 Hybrid topology

In hybrid topology (or model) star, ring and Von Neumann topologies are combined together in the same algorithm. For each generation, the particle will analyze its next position using all different topologies. Particle will select the topology with the smallest fitness value and will update its velocity and position according to it. Modification on basic PSO is going by replacing step 5 in the standard pseudo code, which presented in section 2, by the follows:

```
(5) Analytical steps
(5.1) Star topology evaluation
(5.1.1) temporary Calculate particle
velocity according to equation (1)
and temporary Update particle
position according to equation (2)
(5.2)Ring topology evaluation
(5.2.1) Find lbest using
ring topology (right, left particles)
(5.2.2) temporary Calculate particle
velocity according to equation (3),
and temporary Update particle
position according to equation (2)
(5.3) Von Neumann topology evaluation
(5.3.1)Find lbest using Von Neumann
topology (above, below, right,
and left particles)
(5.3.2) temporary Calculate particle
velocity according to equation (3),
and temporary Update particle
position according to equation (2)
(5.4) calculate fitness for steps
(5.1.2), (5.2.3)  and  (5.3.3)
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(5.5) update velocity and position
using the topology that gave minimum
fitness in step (5.4).
```

5 Benchmark Functions

For comparison, nine benchmark functions that are taken from evolutionary computation literature [17][3][11][4] are used. All functions (except The Camel-back) are high-dimensional problems. Functions Sphere, Schwefel's Problem2.22 and Rosenbrock are unimodal. Step Function has one minimum (unimodal) but it is a discontinuous Function. Schwefel's Problem 2.26, Rastrigin, Ackley and Griwank are multimodal functions where the number of local minimum increases exponentially with problem dimension[11]. For that they appear to be the most difficult class of problems for many optimization algorithms[17]. The Camel-Back function is a low-dimensional function that has only a few local minima.

For each of these functions, the goal is to find the global minimize, formally defined as
$Given f : \Re^{N^d} \rightarrow \Re$
$Find\, x^* \in \Re^{N_d}$ such that $f(x^*) \leq (x), \forall x \in \Re^{N_d}$
The following is functions that were used:

A. *Sphere function*, defined as
$$f_1(x) = \sum_{i=1}^{N_d} x_i^2$$
$where\; x^* = 0$ and $f(x^*) = 0$ for -100$\leq x_i \leq 100$.

B. *Schwefel's Problem 2.22*, defined as
$$f_2(x) = \sum_{i=1}^{N_d} |x_i| + \prod_{i=1}^{N_d} |x_i|$$
$where\; x^* = 0$ and $f(x^*) = 0$ for $-10 \leq x_i \leq 10$.

C. *Step function*, defined as
$$f_3(x) = \sum_{i=1}^{N_d} (\lfloor x_i + 0.5 \rfloor)^2$$
$where\; x^* = 0$ and $f(x^*) = 0$ for -100$\leq x_i \leq 100$.

D. *Rosenbrock function*, defined as
$$f_4(x) = \sum_{i=1}^{N_{d/2}} \left(100\left(x_{2i} - x_{2i-1}^2\right)^2 + (1 - x_{2i-1})^2\right)$$
$where\; x^* = (1, 1, ..., 1)$ and $f(x^*) = 0$ for -30$\leq x_i \leq 30$.

E. *Generalized Swefel's Problem 2.26*, defined as
$$f_5(x) = -\sum_{i=1}^{N_{d/2}} \left(x_i \sin\left(\sqrt{|x_i|}\right)\right)$$
$where\; x^*$=(420,9687,...,420,9687)
and $f(x^*) = -12569.5$ for -500$\leq x_i \leq 500$.

F. *Rastrigin function*, defined as
$$f_6(x) = -\sum_{i=1}^{N_d} \left(x_i^2 - 10\cos(2\pi x_i) + 10\right)$$
$where\; x^*$=(0) and $f(x^*) = 0$ for -5.12$\leq x_i \leq 5.12$.

G. *Ackley's function*, defined as
$$f_7(x) = -20\exp\left(-0.2\sqrt{\frac{1}{30}\sum_{i=1}^{N_d} x_i^2}\right) - \exp\left(\frac{1}{30}\sum_{i=1}^{N_d}\cos(2\pi x_i)\right) + 20 + e$$
$where\; x^*$=(0) and $f(x^*) = 0$ for -32$\leq x_i \leq 32$.

H. *Griewank function*, defined as
$$f_8(x) = \frac{1}{4000}\sum_{i=1}^{N_d} x_i^2 - \prod_{i=1}^{N_d}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$$
$where\; x^*$=(0) and $f(x^*) = 0$ for -600$\leq x_i \leq 600$.

I. *Six-Hump Camel-Back function*, defined as
$$f_9(x) = 4x_1^2 - 21x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$$
$where\; x^*$=(-0.08983,0.7126),(-0.08983,0.7126)
$and\; f(x^*) = -1.0316285$ for -5$\leq x_i \leq 5$.

6 Results and discussion

This section compares the hybrid (combination of star, ring and Von Neumann topologies) PSO to other PSO models. The effect of the number of particles, problem dimension with corresponding number of iterations is explored.

For all the algorithms used in this section, c1 = c2 = 1.49, w = 0.72 and Vmax = half the length of the search space as specified in section 5. The initial population was generated from a uniform distribution in the range specified. Furthermore, for specific run R, the random initialization strategy was used for the first algorithm (i.e. hybrid) only, while all other algorithms (i.e. star, ring and Von Neumann) used the same positions that have been generated for the first algorithm. Hence for comparison purpose the same particles were used in all algorithms in specify run. The velocity was initialized to zero. In order to investigate whether the hybrid PSO scales well or not, different numbers of particles m with different dimensions are investigated. The results reported in this section are averages and standard deviations over 30 simulations. The numbers of particles m that used are 50 and 40. Gmax is set as 500 and 1000 generations corresponding to the dimensions 20 and 30 respectively. Unless otherwise specified.

Tables 1, 2,3,4,5, and 6 list a representative set of results from the conducted experiments. The tables include the test function, the dimensionality of the function, the number of generations the algorithm was run and the average best fitness for the best particle found

Table 1: Mean best fitness of 30 run and standard deviations ($\pm SD$) of uni-modal function optimization results for 50 particles.

f	Dim	Gen	$Hybrid\ topology$	$Star\ topology$	$Ring\ topology$	$Vonv\ Neumann$
	20	500	0.000000±0.000000	0.000000±0.000000	0.000030±0.000026	0.000000±0.000000
$f1$	30	1000	0.000000±0.000000	0.000000±0.000000	0.000000±0.000000	0.000000±0.000000
	20	500	0.000000±0.000000	0.000000±0.000000	0.000684±0.000262	0.000014±0.000005
$f2$	30	1000	0.000000±0.000000	0.000000±0.000000	0.000017±0.000007	0.000000±0.000000
	20	500	0.000000±0.000000	0.000000±0.000000	0.000000±0.000000	0.000000±0.000000
$f3$	30	1000	0.000000±0.000000	0.033333±0.182574	0.000000±0.000000	0.000000±0.000000
	20	500	**13.611410±7.903354**	26.732893±31.441795	19.478951±13.006731	23.527544±22.735853
$f4$	30	1000	**24.534685±18.997162**	155.183746±565.559387	34.818619±26.005678	44.442329±46.125607

Table 2: Mean best fitness of 30 run and standard deviations ($\pm SD$) of uni-modal function optimization results for 40 particles.

f	Dim	Gen	$Hybrid\ topology$	$Star\ topology$	$Ring\ topology$	$Vonv\ Neumann$
	20	500	0.000000±0.000000	0.000000±0.000000	0.000035±0.000026	0.000000±0.000000
$f1$	30	1000	0.000000±0.000000	0.000000±0.000000	0.000000±0.000000	0.000000±0.000000
	20	500	0.000000±0.000000	0.000000±0.000000	0.000728±0.000304	0.000022±0.000010
$f2$	30	1000	0.000000±0.000000	0.000000±0.000000	0.000017±0.000007	0.000000±0.000000
	20	500	0.000000±0.000000	0.000000±0.000000	0.000017±0.000006	0.000000±0.000000
$f3$	30	1000	0.000000±0.000000	0.133333±0.730297	0.000000±0.000000	0.000000±0.000000
	20	500	**11.569695±3.735688**	26.766483±39.607185	35.082821±25.284101	25.695148±23.108534
$f4$	30	1000	**35.504734±28.051165**	42.174423±28.117460	41.780052±24.425417	38.154171±25.684706

Table 3: Mean best fitness of 30 run and standard deviations ($\pm SD$) of uni-modal function optimization results for 50 particles, Dimension = 10 and Gmax=1000 .

f	Dim	Gen	$Hybrid\ topology$	$Star\ topology$	$Ring\ topology$	$Vonv\ Neumann$
$f1$	10	1000	0.000000±0.000000	0.000000±0.000000	0.000000±0.000000	0.000000±0.000000
$f2$	10	1000	0.000000±0.000000	0.000000±0.000000	0.000017±0.000006	0.000000±0.000000
$f3$	10	1000	0.000000±0.000000	0.000000±0.000000	0.000000±0.000000	0.000000±0.000000
$f4$	10	1000	**0.290909±1.018993**	0.871010±1.341269	2.073656±1.536980	1.778074±0.814718

for the 30 runs of the test functions respectively. Standard deviation for each value is also listed.

Tables also list the corresponding average best fitness for the Standard PSO using Star topology, Ring topology and Von Neumann topology with the same settings (where they are applicable) as described in the previous section. Unimodal functions are listed in Table 1, 2 and 3. Where Multimodal Functions are represented in Tables 4, 5 and 6.

6.1 Unimodal Functions

Table 1, Table 2 and Table 3, summarize the results obtained by applying the different methods to the unimodal problems. The results show that hybrid PSO performed better than (or at least equal to) the other strategies in all tested functions.

By experiments, the best results for unimodel functions have been obtained when the population size set to 50, dimension of functions set to 10 and the iterations set to 1000 as showing in Table 3. Figure 2 shows the average best fitness for each generation (Gmax=1000) for tested unimodal functions. The figures compare Hybrid topology model with other versions of PSO. In graphs hybrid refer to PSO using hybrid of star, ring and VN topologies, gbest refer to standard PSO using star topology, lbest refer to standard PSO using ring topology, where Von Neumann refer to standard PSO using Von Neumann topology. The graphs illustrate a representative set of experiments for functions using 50 particles with a dimension set to 30.

In experiments with the four unimodal functions, the hybrid PSO achieved better results and had the faster convergence rate than all other versions of PSO. Albeit the convergence speed for gbest model-star topology-is close to convergence speed of hybrid model, gbest model trapped in a local optimum for the Rosenbrock function very early during the search. The Ring topology has the slowest conversion speed.

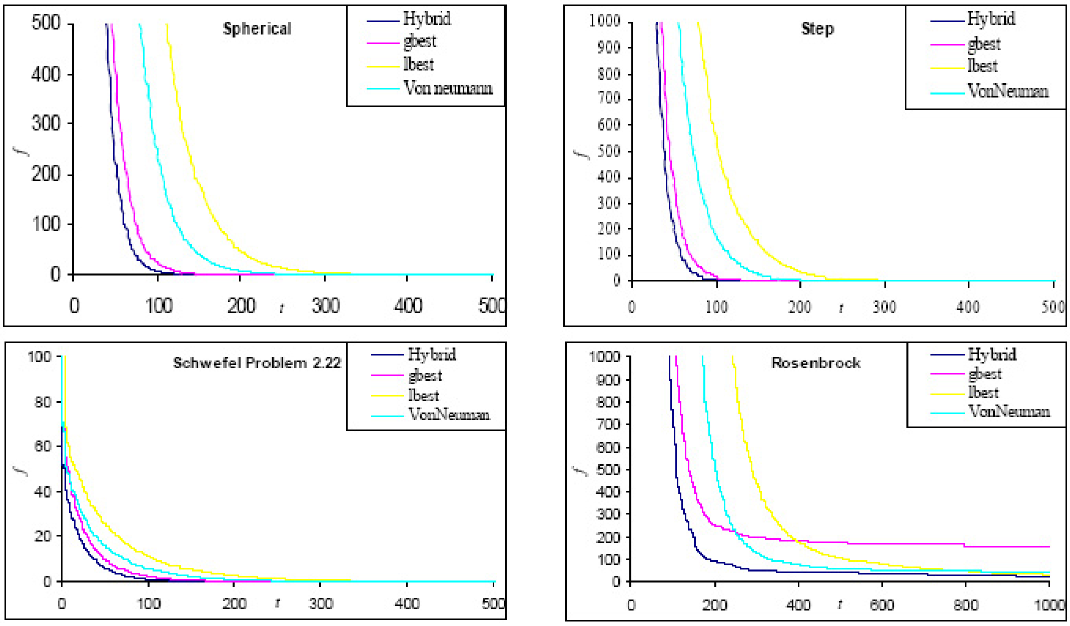

Figure 2: Comparison between the different versions of PSO on the unimodal functions. The vertical axis represents the function value and the horizontal axis represents the number of generations.

Table 4: Mean best fitness of 30 run and standard deviations ($\pm SD$) of multi-modal function optimization results for 50 particles.

f	Dim	Gen	$Hybrid\ topology$	$Star\ topology$	$Ring\ topology$	$Vonv\ Neumann$
$f5$	20	500	**-6582.76807±386.52445**	-5996.71143±524.58221	-5866.97998±330.94025	-6059.55469±357.65851
	30	1000	**-8914.60547±725.81543**	-8422.02441±645.38501	-8096.08887±486.61813	-8526.37114±699.16046
$f6$	20	500	**15.072241±5.235845**	24.13549±10.242969	23.827868±6.161203	18.37709±6.360409
	30	1000	**29.580095±7.421592**	47.109001±14.644919	43.183247±9.907544	34.49754±8.657301
$f7$	20	500	**-0.000002±0.000000**	0.213074±0.569593	0.002532±0.001336	0.000061±0.000033
	30	1000	**-0.000002±0.000000**	0.908264±0.869754	0.000107±0.000056	-0.000001±0.000000
$f8$	20	500	0.020594±0.014840	0.021555±0.018157	**0.002014±0.004293**	0.010821±0.013712
	30	1000	0.007554±0.007312	0.017034±0.030371	**0.001738±0.004388**	0.006651±0.009345
$f9$	2	500	-1.031628±0	-1.031628±0	-1.031628±0	-1.031628±0
	2	1000	-1.031628±0	-1.031628±0	-1.031628±0	-1.031628±0

Table 5: Mean best fitness of 30 run and standard deviations ($\pm SD$) of multi-modal function optimization results for 40 particles.

f	Dim	Gen	$Hybrid\ topology$	$Star\ topology$	$Ring\ topology$	$Vonv\ Neumann$
$f5$	20	500	**-6234.282227±465.074768**	-5854.051758±580.406738	-5589.804199±367.290497	-5946.909180±604.808838
	30	1000	**-9093.771484±627.405029**	-8136.155762±596.664673	-8148.088379±634.528687	-8549.250977±639.801392
$f6$	20	500	**17.064165±5.176713**	24.899055±9.928655	25.317053±4.282255	20.124708±6.895897
	30	1000	**34.526691±8.480634**	53.018333±15.796675	43.660721±9.189561	35.492195±9.290305
$f7$	20	500	**-0.000002±0.000000**	0.344388±0.642362	0.002796±0.001943	0.000073±0.000031
	30	1000	**-0.000002±0.000000**	1.377343±0.861749	0.000178±0.000190	-0.000001±0.000001
$f8$	20	500	0.017056±0.015856	0.029470±0.030515	0.010097±0.011802	0.011811±0.014860
	30	1000	0.005994±0.007162	0.014171±0.014704	0.001504±0.003455	0.005247±0.010582
$f9$	2	500	-1.031628±0	-1.031628±0	-1.031628±0	-1.031628±0
	2	1000	-1.031628±0	-1.031628±0	-1.031628±0	-1.031628±0

6.2 Multimodal functions

Multimodal functions with many local optima are often difficult to optimize. Table 4, Table 5 and Table 6, sum-

Table 6: Mean best fitness of 30 run and standard deviations ($\pm SD$) of multi-modal function optimization results for 50 particles, Dimension=10 and Gmax=1000.

f	Dim	Gen	$Hybrid\ topology$	$Star\ topology$	$Ring\ topology$	$Vonv\ Neumann$
$f5$	10	1000	**-3636.255371±208.135071**	-3397.954590±254.216766	-3426.145752±263.891815	-3599.254395±233.310791
$f6$	10	1000	**2.722301±1.500988**	6.440560±4.133302	5.246451± 1.956245	3.463268±1.319513
$f7$	10	1000	**-0.000002±0.000000**	-0.000002±0.000000	-0.000002± 0.000000	-0.000002±0.000000
$f8$	10	1000	0.043794±0.022501	0.072239±0.040020	**0.028187±0.019831**	0.032108±0.020075
$f9$	2	1000	-1.031628±0.000000	-1.031628±0.000000	-1.031628±0.000000	-1.031628±0.000000

marizes the results obtained by applying the different methods to the multimodal problems.

The results show that the hybrid PSO performed better than (or at least equal to) the other strategies in all the tested functions except for the Griewank function, where lbest using ring topology performed slightly better. Table 4, 5 and 6 show that in general the performance of Schwefel Problem 2.26 function and Griewank function improved by increasing the dimension problem. While other tested functions performed better when the population size is increased and dimension problem is decreased.

Figure 3 shows the average best fitness for each generation (Gmax=1000) for multimodal function (Camelback not included). The graphs illustrate a representative set of experiments for functions using 50 particles with a dimension set to 30.

7 Conclusion and Future Work

In this paper a hybrid model of PSO based on the multi types of neighbourhood topologies was introduced. The hybrid model was basically the standard PSO working on multi types of neighbourhood topologies. In standard model the follow of information between particles depend on one type of topology. The global best model uses the Star topology, and the local model uses either the ring or the Von Neumann topologies. The hybrid model presented in this paper has the ability to work with three types of topologies, star, ring and Von Neumann. The purpose of that was to get the benefit of strength of using global best and local best models. The performance of the proposed algorithm was compared to other standard types of PSO with a set of unimodal functions and multimodal functions. The results showed that, the new algorithm almost performed better fitness and has faster convergence speed comparing to other strategies.

However by recording the selected topology for some particles through 1000 generation - find Figure 4 to 7-, the results showed that, the particle prefers to use the star topology -gbest model- most of the time, particularly within first iterations. Unfortunately this may weak the new strategy, especially if the particles trapped in local minimum in first iterations. Future works will investigate this problem and will attempt to solve it. Future work would be built on sequential hybring. For example, the hybrid algorithm might be improved by imposing the particles to use the lbest model in first iterations. Hence more search area will be investigated to not trap in local minimum. Furthermore, other values of C1, C2, w, population size and dimensional problems should be investigated and compared to other algorithms.

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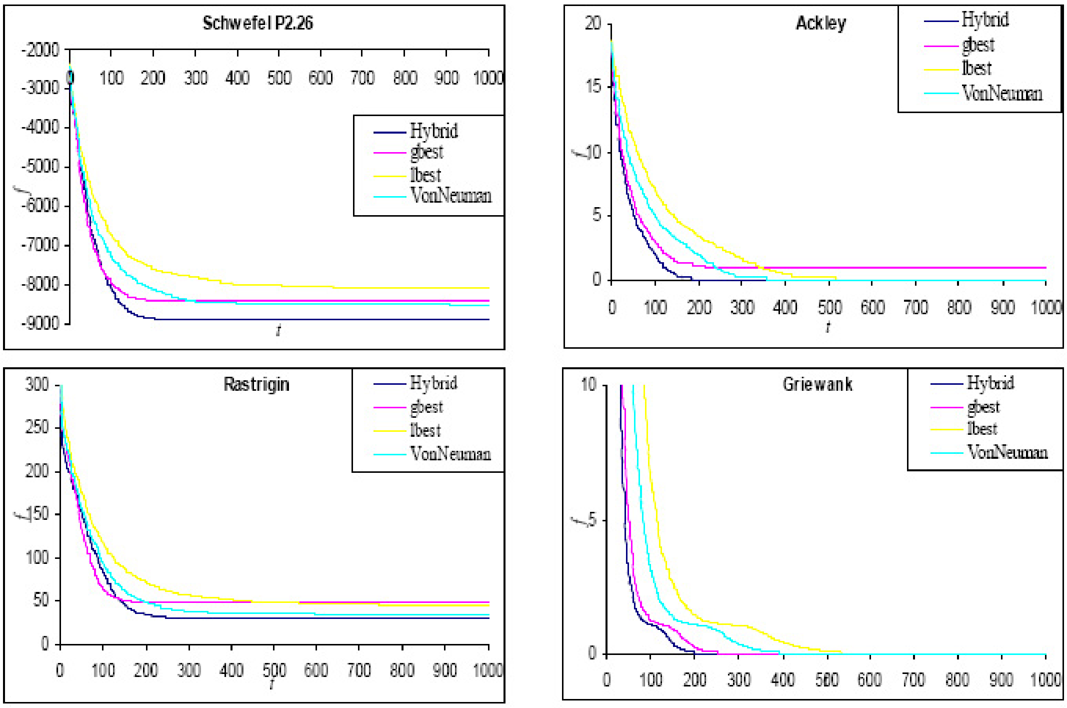

Figure 3: Comparison between the different versions of PSO on the Multimodal functions. The vertical axis represents the function value and the horizontal axis represents the number of generations.

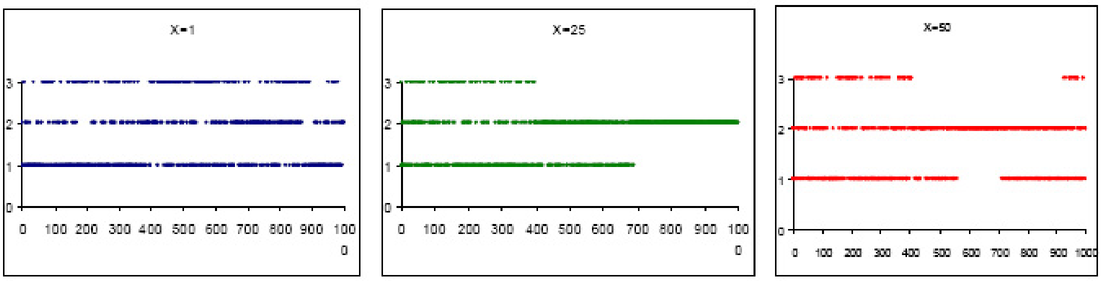

Figure 4: Represented the preferred topology for some particles (x) in last run for **Rosenbrock function**. The vertical axis represents the selected topology 1 represents 'Star', 2 represents 'Ring', 3 represents 'VN' and the horizontal axis represents the number of generations

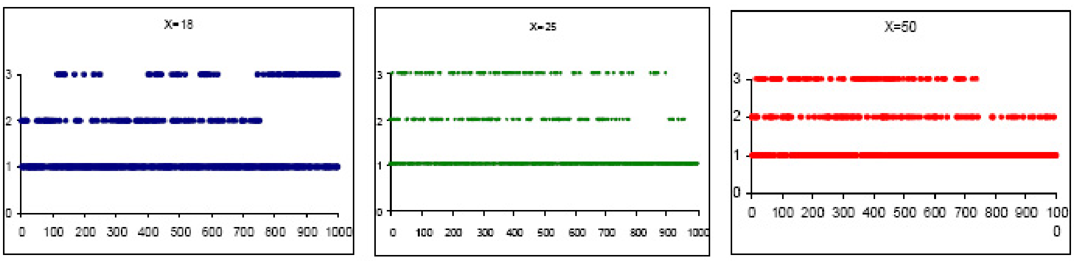

Figure 5: Represented the preferred topology for some particles (x) in last run for **Rastrigin function**. The vertical axis represents the selected topology (1 represents 'Star', 2 represents 'Ring', 3 represents 'VN') and the horizontal axis represents the number of generations.

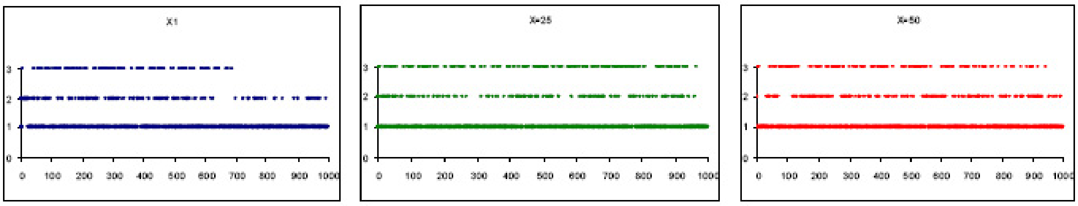

Figure 6: Represented the preferred topology for some particles (x) in last run for **Ackley function**. The vertical axis represents the selected topology (1 represents ‘Star’, 2 represents ’Ring’, 3 represents ’VN’) and the horizontal axis represents the number of generations.

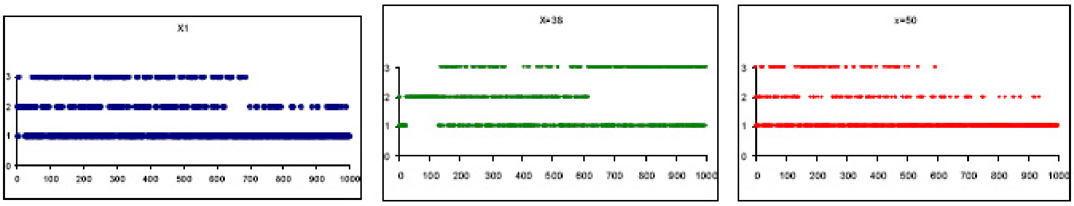

Figure 7: Represented the preferred topology for some particles(x) in last run using **Schwefel’s Problem 2.26 function**. The vertical axis represents the selected topology (1 represents ’Star’, 2 represents ’Ring’, 3 represents ’VN’) and the horizontal axis represents the number of generations.

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