# Multilevel Thresholding based on Fuzzy C Partition and Gravitational Search Algorithm

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**Abstract.** Entropy based image thresholding methods are widely adopted for multilevel image segmentation. Bilevel thresholding partitions an image into two classes, whereas multilevel thresholding partitions an image into multiple classes depending upon thresholding level . The automatic selection of optimal threshold is often treated as an optimization problem. This paper contributes to novel thresholding method, that is based on entropy of fuzzy c partition and gravitational search algorithm (GSA). Experiments have been evaluated on the different test images and results were assessed by entropy, stability, computation time and peak signal to noise ratio (PSNR). The analysis of results conveys that the GSA outperform particle swarm optimization (PSO).

**Keywords:** Fuzzy c partition, Thresholding, Image segmentation, GSA, PSO.

## 1  Introduction

Image segmentation is an important processing stage in image, video and computer vision application. It is an image analysis step in which input are images and output are features extracted from those images. It splits an image into significant regions or objects depending on the particular application. It basically distinguishes objects from the background on the basis of color, intensity value of pixels etc. Often, in numerous applications the intensity value of the pixels belonging to the object are greatly different from pixels those belong to background. In such circumstances, thresholding act as a simple but effective tool for separating objects from the background. It is an important issue to search accurate threshold value that separate different objects from background. Though, the automatic selection of optimal threshold has remained a challenge in image segmentation.

Many approaches that are recommended in the literature for thresholding may be seen as: (i) Histogram based methods, (ii) Entropy based methods, (iii) Clustering based methods, (iv) Higher order statistics based methods and (v) Local characteristics based methods [13]. Of specific interest is the concept of entropy that was introduced by Shannon in information theory. It was introduced into thresholding based segmentation by Pun [10], whose work was based on the principle of maximum entropy. Kapur et al. [8] worked on the shortcoming of Pun's method and developed a method for selection of optimum threshold which maximizes the sum of the entropy of the segmented regions i.e. the object region and background region.

It is assumed that there exists some fuzziness and vagueness in images. So, fuzzy theory is applied to deal with this ambiguity and proves to be effective technique. Fuzzy c partition is widely used approach to measure the amount of fuzziness in the images[2, 5, 14, 15].Cheng et al.[4], presented fuzzy homogeneity vectors to handle the grayness and spatial uncertainties among pixels to perform multilevel thresholding. In [5],

Cheng et al. used the concept of fuzzy c partition and the maximum entropy principle to select threshold values for gray-level images. They used exhaustive approach for bilevel thresholding for the searching of optimal combination of fuzzy parameters while using simulated annealing for the multilevel thresholding to reduce computational complexity. In [6] a new approach for thresholding was presented that involve fuzzy partitioning on two-dimensional histogram using the principle of entropy and genetic algorithm to determine the optimal thresholds. Zhao et al. [16] designed three-level thresholding method which is based on the correlation between the fuzzy c partition, the probability partition and maximum entropy theory. They utilized genetic algorithm to obtain the optimum fuzzy parameters. Tao et al. [15] presented three-level thresholding method using concept of fuzzy entropy through probability analysis. The image is divided into three fuzzy regions determined by membership function. So, in order to find out the six parameters in those membership functions, they adopted genetic algorithm to estimate the optimal combination of the fuzzy parameters. Tao et al. [14] extended their work in fuzzy entropy and used the ant colony optimization (ACO) to obtain the optimal parameters and compared the results with the existing methods. Assas & Benmahammed [2] performed the particle swarm optimization (PSO) in order to reduce the time required to obtain thresholding value for multilevel thresholding.

In this paper, our prime goal is to determine a threshold and multiple thresholds for bilevel and multilevel thresholds respectively for the image segmentation. To obtain the accurate threshold values, it is necessary to obtain the optimal fuzzy parameters that define the fuzzy partition. But the computational complexity increases exponentially when fuzzy regions are calculated using fuzzy parameters through the exhaustive search. To remove above issues and for fast thresholding, metaheuristic optimization techniques have been applied to multilevel thresholding [1, 12]. Metaheuristic optimization provides a common framework for solving a problem. Various stochastic metaheuristic algorithms such as simulated annealing, genetic, ant colony optimization etc. have been applied to multilevel thresholding using fuzzy entropy [5, 14, 15].

To determine the optimal set of fuzzy parameters effectively, this paper utilizes the recently developed gravitational search algorithm (GSA). It is one of the recent heuristic optimization that was introduced by Rashedi et al.[11]. It is new stochastic optimization technique that uses the concept of physical phenomena i.e. law of gravity. The group of masses attracted towards the large mass, which leads to optimum solution. The feasibility of the proposed method is demonstrated on six different real images and compared with particle swarm optimization (PSO). The results show that the proposed method for thresholding can significantly outperform on the basis of the solution quality, stability and objective function. Multilevel thresholding provides much meaningful information as compared to bilevel thresholding. Thus, multilevel thresholding plays significant role in computer vision. Current literature does not address generalized multilevel thresholding for fuzzy entropy. This paper generalizes multilevel thresholding for the same.

The paper is organized as follows: Section 2 formulates the problem of multilevel thresholding. The overview of gravitational search algorithm (GSA) is described in Section 3. In Section 4, proposed approach using GSA is presented. The performance of proposed method is validated over wide range of images and results are discussed in Section 5 . Finally, the conclusion is given in Section 6.

## 2  Problem formulation based on Fuzzy c partition

With concern towards optimal threshold selection in segmentation, the theory of fuzzy entropy proves to be influential and proficient one. Images are represented as fuzzy c partition that is determined by membership function.

Suppose L be the gray levels in an image and these gray levels are in the range 0, 1, 2... (L-1). The probability of gray level i can be described by Eq.1

$$p_i = \frac{h(i)}{N} \qquad (1)$$

where h(i) represents number of occurrence of each gray level i and N is total number of pixels in an image . The probability occurrence of a fuzzy set A is given by Eq.2 :

$$P(A) = \sum_{i=0}^{L-1} \mu_{A_c} p_i \qquad (2)$$

Our objective function is to maximize entropy of the fuzzy c partition which is obtained by Eq.3

$$H = -\sum_{j=1}^{c} P(A)_j \log P(A)_j \qquad (3)$$

where c is the number of fuzzy partitions or sets.

## 2.1  Bilevel Thresholding

Bilevel thresholding classifies the pixels into two classes, one including those pixels with gray levels above a certain threshold and the other including the rest. For this purpose, we need a single threshold value. Consider two fuzzy sets $A_1$ and $A_2$ whose membership function is given by Eq.45 [5]:

$$\mu_{A_1}(i) = \begin{cases} 1 & i \le a_1 \\ \dfrac{i - b_1}{a_1 - b_1} & a_1 < i \le b_1 \\ 0 & b_1 > i \end{cases} \tag{4}$$

$$\mu_{A_2}(i) = \begin{cases} 0 & i \le a_1 \\ \dfrac{i - a_1}{b_1 - a_1} & a_1 < i \le b_1 \\ 1 & b_1 > i \end{cases} \tag{5}$$

where i is gray level varies from 0 to L-1 and $a_1$, $b_1$ are the fuzzy parameters required to define the membership function. The value of threshold is obtained by taking midpoint of fuzzy parameters $a_1$, $b_1$.

## 2.2  Multilevel Thresholding

Multilevel thresholding partitions the pixels into several classes. The pixels belonging to the same class have gray levels within a specific range defined by multiple thresholds. Here, the membership function for c-level thresholding is presented. For fuzzy c partitions, membership function is defined by Eq.678 :

$$\mu_{A_1}(i) = \begin{cases} 1 & i \le a_1 \\ \dfrac{i - b_1}{a_1 - b_1} & a_1 < i \le b_1 \\ 0 & b_1 > i \end{cases} \tag{6}$$

$$\mu_{A_{c-1}}(i) = \begin{cases} 0 & i \le a_{c-2} \\ \dfrac{i - a_{c-2}}{b_{c-2} - a_{c-2}} & a_{c-2} < i \le b_{c-2} \\ 1 & b_{c-2} < i \le a_{c-1} \\ \dfrac{i - b_{c-1}}{a_{c-1} - b_{c-1}} & a_{c-1} < i \le b_{c-1} \\ 0 & b_{c-1} > i \end{cases} \tag{7}$$

$$\mu_{A_c}(i) = \begin{cases} 0 & i \le a_{c-1} \\ \dfrac{i - a_{c-1}}{b_{c-1} - a_{c-1}} & a_{c-1} < i \le b_{c-1} \\ 1 & b_{c-1} > i \end{cases} \tag{8}$$

where i varies from 0 to L-1 and $a_1$, $b_1$ ..$a_{c-1}$, $b_{c-1}$ are the fuzzy parameters which should follow the condition $0 \le a_1 < b_1 <,..,< a_{c-2} < b_{c-2} < a_{c-1} < b_{c-1} \le L - 1$ to calculate the c-1 threshold values to partition image into c classes. Hence, fuzzy c partition can be determined by 2(c-1) fuzzy parameters. So, the problem becomes to determine the optimal combination of these fuzzy parameters such that the entropy of the image is maximized.

## 3  Overview of Gravitational search algorithm

Gravitational search algorithm (GSA) is designed by Rashedi et al. [11] . It is theoretical substitution of Newton's two laws. First one is the gravitational law which states that every particle in the universe attract other particles with force that is directly proportional to product of their masses and inversely proportional to distance between them. Second one is Newton's II law of motion which states that acceleration of an agent depends on its mass and total force applied on it. Attraction exists between all the masses, but the weaker ones are well pulled by heavy mass. Heavy mass, here represents efficient agent as it has better attraction capability which leads to better outcomes. Detailed description of the GSA is given below [11]:

Each agent has gravitational mass which is calculated using fitness value is given by Eq.910

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \tag{9}$$

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \tag{10}$$

where $fit_i$ is fitness value of agent i and worst and best are the worst and best fitness value among all the agents. The position of each agent represents solution for problem domain which is defined by Eq.11:

$$X_i^D =_1^D, x_2^D, ..., x_N^D\} \tag{11}$$

where D is the dimension of the agent. The velocity and position of an agent for next iteration is calculated by Eq.12 and Eq.13 respectively.

$$v_i^D(t+1) = r_i v_i^D(t) + a_i^D(t) \tag{12}$$

$$x_i^D(t+1) = x_i^D(t) + v_i^D(t+1) \tag{13}$$

where r is random number which lies between [0-1]. The acceleration of the agent i by Newtons II law of motion is obtained by Eq.14

$$a_i^D(t) = \frac{F_i^D(t)}{M_i(t)} \tag{14}$$

The total force $F_i$ applied on agent i by other agents is given by Eq.15

$$F_i^D(t) = \sum_{j \in Kbest, j \neq i}^{N} r_j F_{ij}^D(t) \qquad (15)$$

where N represents number of agents, $r_j$ is random number that lies between 0 to 1 and Kbest are those agents which are heavier. In the beginning all agents belong to Kbest but after each step, value of Kbest decreases and at the end of the algorithm we will have only one agent that is the optimal solution to the problem. Force exerted on mass $M_i$ by mass $M_j$ that having distance $R_{ij}$ between them is obtained by Eq.16

$$F_{ij}^D(t) = G(t) \frac{M_i(t)M_j(t)}{R_{ij}(t) + \varepsilon}(x_i^D - x_j^D) \qquad (16)$$

where G(t) is gravitational constant. It is used to limit the search space and it is the function of time which is decreased after every iteration is given by Eq.17.

$$G(t) = G_0(t) \times (\frac{t}{T})^\beta \qquad (17)$$

$\beta < 1$ and T is total number of iteration.

## 4  Proposed Approach

The problem is to segment an image by multilevel thresholding such that the entropy of the image is maximized. We can rewrite objective function (entropy) which is to be maximized by Eq.18

$$H = \sum_{j=1}^{c}(\sum_{i=0}^{L-1} \mu_{A_c} p_i \log(\sum_{i=0}^{L-1} \mu_{A_c} p_i)) \qquad (18)$$

For above objective function, constraint is defined in terms of fuzzy parameter. Fuzzy parameter $a_j$, $b_j$ defines shape of fuzzy membership function, where j is defined as $1 < j \leq c$. Fuzzy parameters should satisfy the following conditions:$0 \leq a_1 < b_1 <, .., < a_{c-2} < b_{c-2} < a_{c-1} < b_{c-1} \leq L - 1$.

In the proposed approach, GSA is employed to determine the pairs of fuzzy parameters a, b that maximize the entropy of fuzzy c partition. The optimal threshold value is obtained by taking midpoint of these pair $(a_j, b_j)$ [5]. The proposed algorithm using GSA is given as:

a) Take image as input and calculate the histogram of the image.

b) Compute the probability of occurrence of gray level P(i) for i = 0 to L-1 gray level.

c) Randomly initialize the initial values such as agent's position, velocity, dimension i.e. the number of fuzzy parameters that depends on the threshold used for segmentation, $G_0$=100,$\beta = 0.2$.

d) For each agent, calculate the fitness function i.e. entropy H for fuzzy c partition. If fitness function is better than previous then store the position of agents as the fuzzy parameters.

e) Calculate the mass M of each agent by Eq.9 and update the gravity G by Eq.17

f) Calculate the total force exerted on each agent by Eq.15 and acceleration by Eq.14

g) Update the velocity and position of an agent by Eq.12 and Eq.13 respectively.

h) Repeat steps from d) to g) till the algorithm reaches the point of fixed number of iteration or maximum entropy.

i) The required c-1 threshold values are calculated using pair of fuzzy parameters $(a_1, b_1)$ .. $(a_{c-1}, b_{c-1})$ by Eq.19:

$$t_1 = \frac{1}{2}(a_1 + b_1), .., t_{c-1} = \frac{1}{2}(a_{c-1} + b_{c-1}) \quad (19)$$

Flowchart of the proposed algorithm using GSA is presented in Figure 8.

## 5  Experimental Results

The experiments were carried on Intel core i5 platform with a 2.5 GHz processor and 4 GB memory running under the Windows 7 operating system. Matlab version 7 was used as simulation software. The performance of the proposed method has been tested on the 2 well known images lena, pepper and 4 other images 12003, 23084, 376001, 113016 from the Berkeley Segmentation Dataset and Benchmark. Each image is presented with its histogram along with its segmented image using 2-level, 3-level, 4-level thresholding from Figures 1-6. In each figure from Figures 1-6 (1a) represent the histogram of the original image (1b) represent the original image (2a) Bilevel segmented image (2b) Membership function of fuzzy 2 partition (3a) 3-level segmented image (3b) Membership function of fuzzy 3 partition (4a) 4-level segmented image (4b) Membership function of fuzzy 4 partition.

The quality of the solution of the method that utilized the GSA and the other that employed PSO are compared on the basis of the value of the best fitness i.e. entropy of fuzzy partition. Of course, larger the
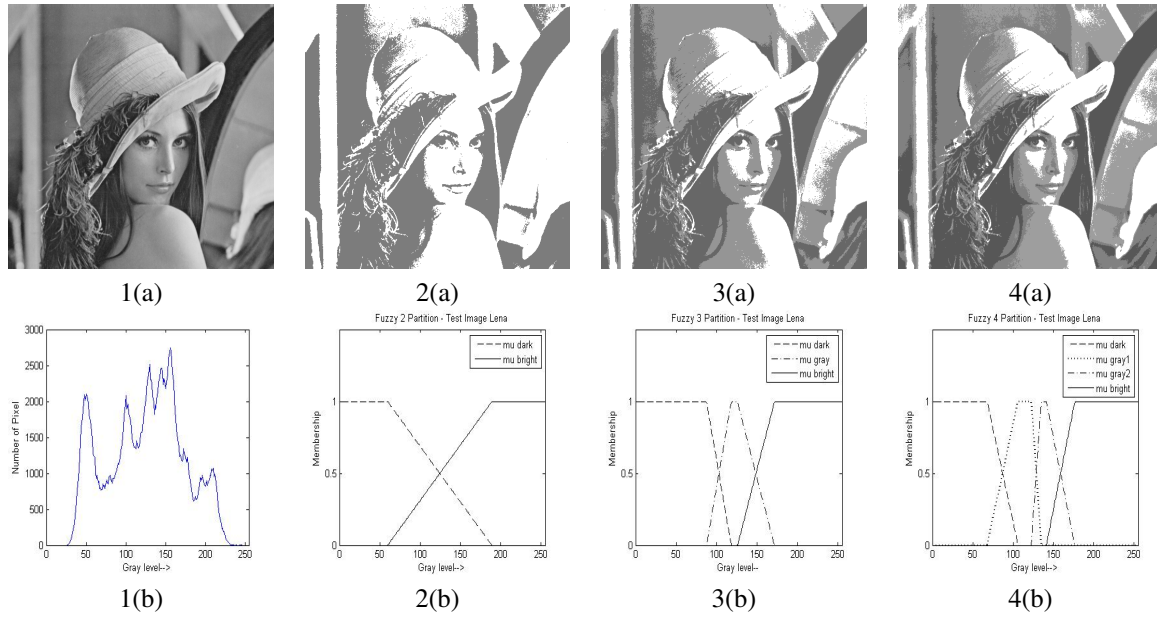
**Figure 1:** 1(a) Original Image: Lena 1(b) Histogram 2(a) Segmented image using bilevel thresholding 2(b) Fuzzy 2 partition 3(a) Segmented using 3 level thresholding 3(b) Fuzzy 3 partition 4(a) Segmented image using 4 level thresholding 4(b) Fuzzy 4 partition



**Figure 2:** 1(a) Original Image: Pepper 1(b) Histogram 2(a) Segmented image using bilevel thresholding 2(b) Fuzzy 2 partition 3(a) Segmented using 3 level thresholding 3(b) Fuzzy 3 partition 4(a) Segmented image using 4 level thresholding 4(b) Fuzzy 4 partition

1(a)          2(a)          3(a)          4(a)

1(b)          2(b)          3(b)          4(b)

**Figure 3:** 1(a) Original Image: 12003 1(b) Histogram 2(a) Segmented image using bilevel thresholding 2(b) Fuzzy 2 partition 3(a) Segmented using 3 level thresholding 3(b) Fuzzy 3 partition 4(a) Segmented image using 4 level thresholding 4(b) Fuzzy 4 partition
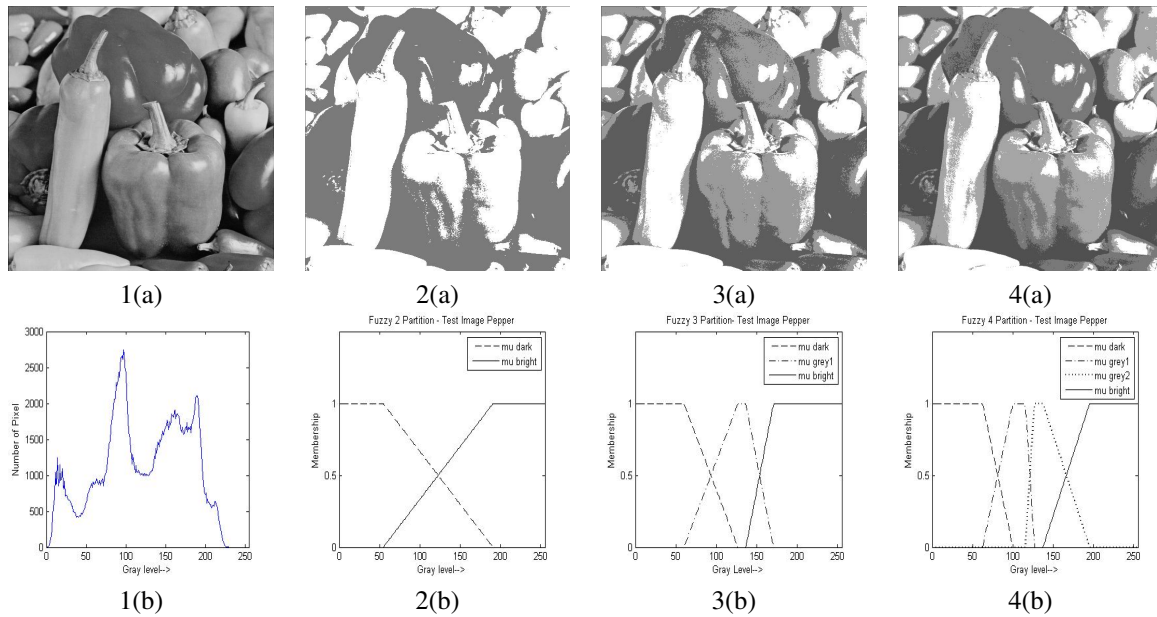


1(a)          2(a)          3(a)          4(a)

1(b)          2(b)          3(b)          4(b)

**Figure 4:** 1(a) Original Image: 23084 1(b) Histogram 2(a) Segmented image using bilevel thresholding 2(b) Fuzzy 2 partition 3(a) Segmented using 3 level thresholding 3(b) Fuzzy 3 partition 4(a) Segmented image using 4 level thresholding 4(b) Fuzzy 4 partition
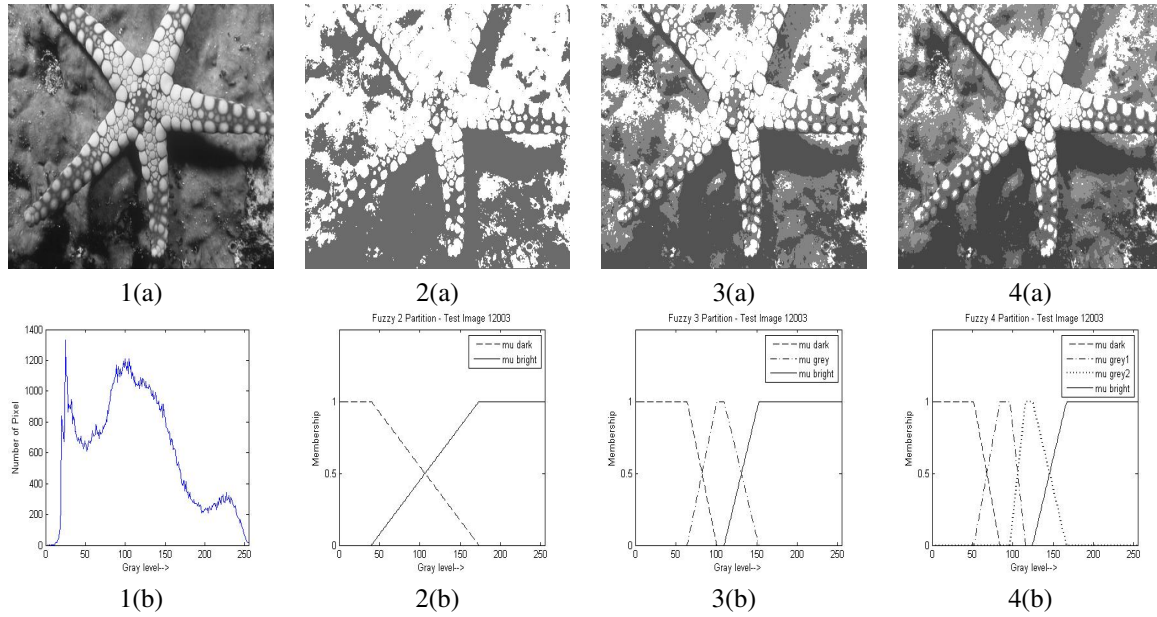
1(a)                2(a)                3(a)                4(a)



1(b)                2(b)                3(b)                4(b)

**Figure 5:** 1(a) Original Image: 376001 1(b) Histogram 2(a) Segmented image using bilevel thresholding 2(b) Fuzzy 2 partition 3(a) Segmented using 3 level thresholding 3(b) Fuzzy 3 partition 4(a) Segmented image using 4 level thresholding 4(b) Fuzzy 4 partition
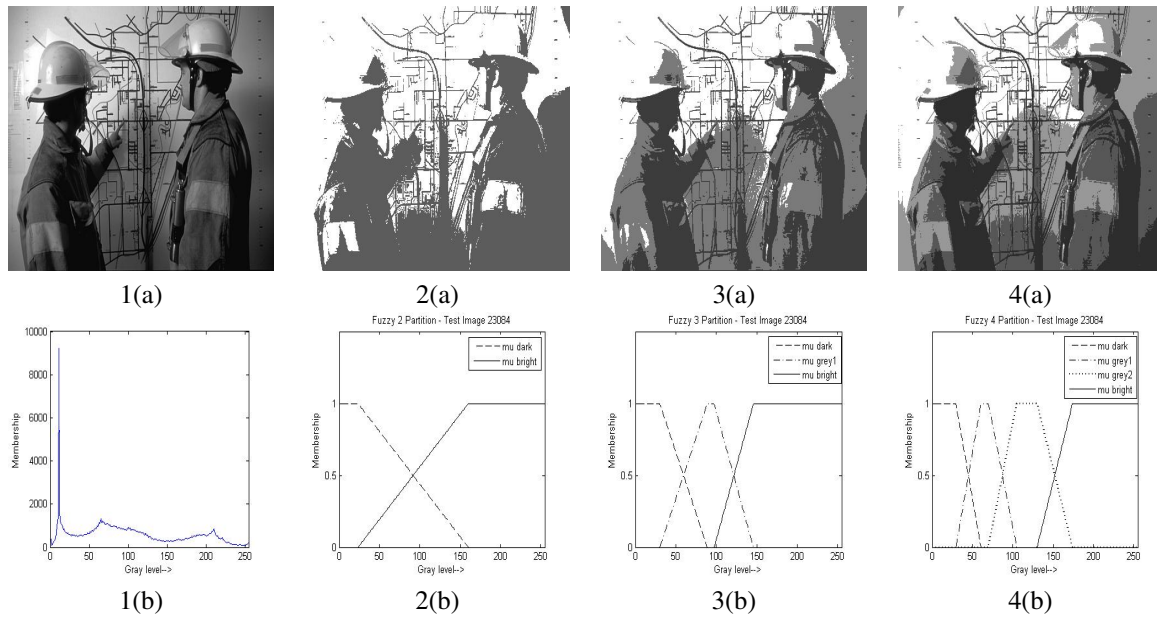


1(a)                2(a)                3(a)                4(a)



1(b)                2(b)                3(b)                4(b)

**Figure 6:** 1(a) Original Image: 113016 1(b) Histogram 2(a) Segmented image using bilevel thresholding 2(b) Fuzzy 2 partition 3(a) Segmented using 3 level thresholding 3(b) Fuzzy 3 partition 4(a) Segmented image using 4 level thresholding 4(b) Fuzzy 4 partition
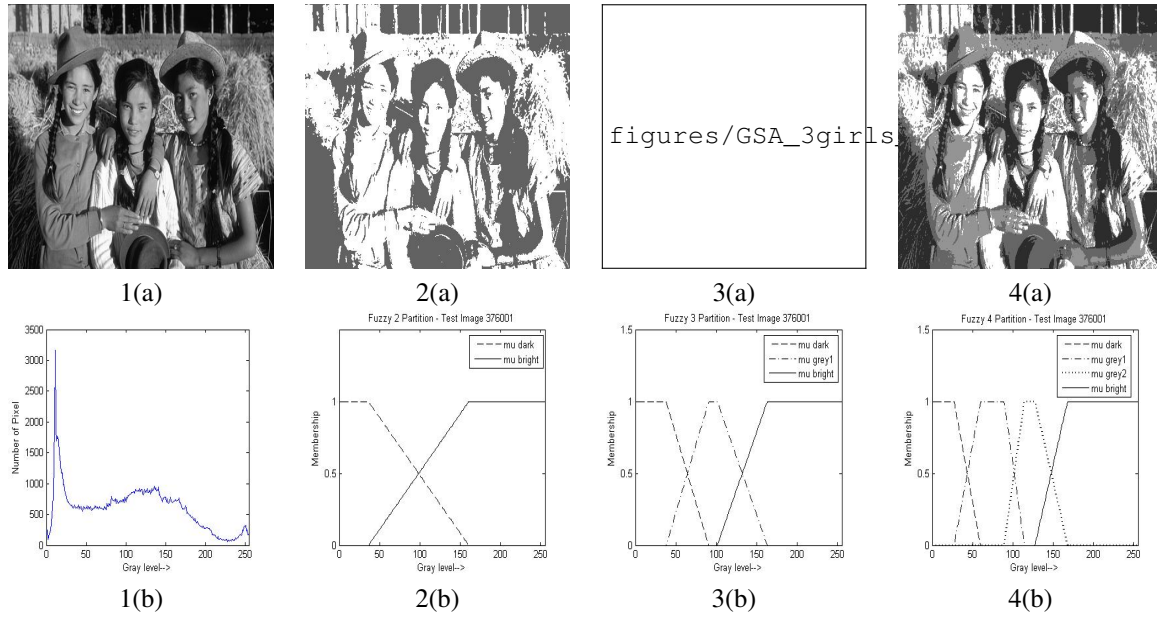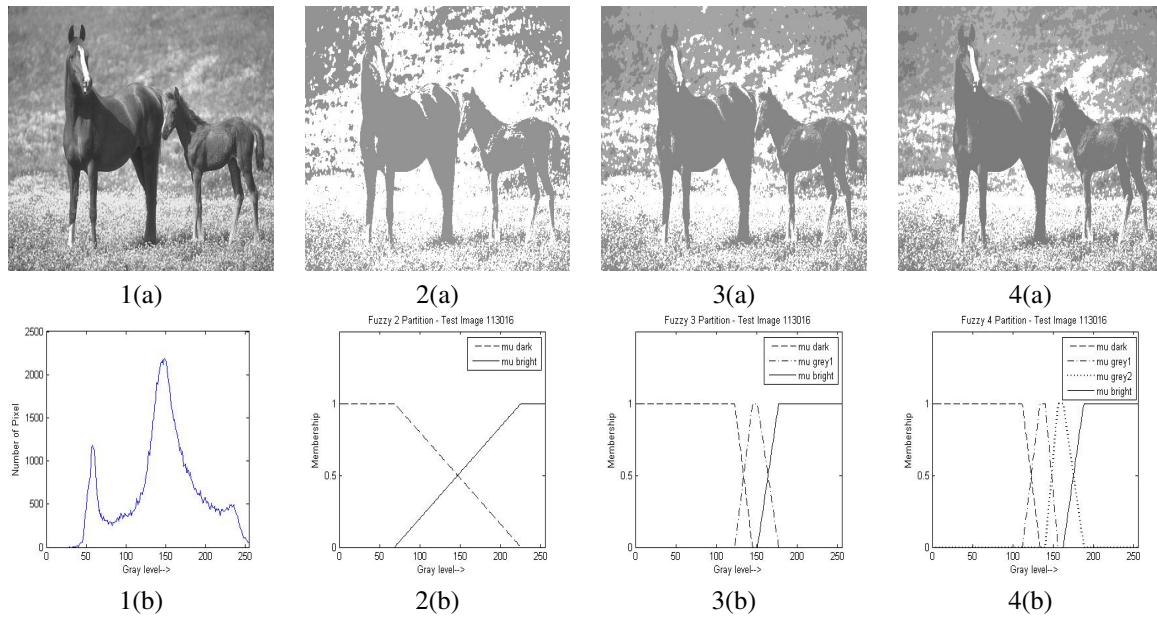
objective function or fitness value, better the algorithm. In addition to this, results are also compared on the parameters such as peak signal to noise ratio (PSNR) and standard deviation for the visual quality and stability of the algorithm respectively.

PSNR is used as quality measurement between original image and processed image. PSNR provides the information of similarity of an image against original image based on the rooted mean square error (RMSE) of each pixel which is defined by Eq.20

$$RMSE = \sqrt{\frac{\sum_{i=1}^{X}\sum_{j=1}^{Y}(I(i,j) - I_{new}(i,j))^2}{X * Y}}$$

(20)

where $I$ and $I_{new}$ are original image and segmented image of size $X \times Y$ respectively. PSNR is measured in decibel (dB), is given by Eq.21.

$$PSNR = 20\log\frac{255}{RMSE}$$

(21)

Standard deviation of the algorithm is calculated for analyzing the stability of the algorithm. Lower the value of the standard deviation, higher the stability of the algorithm. It is given by Eq.22

$$standard\_deviation = \sqrt{\frac{\sum_{i=1}^{m}\sigma_i - \mu}{m}}$$

(22)

where m is the total number of runs to which experiment is repeated (m=30 is used), $\sigma_i$ is the best objective value of $i^{th}$ run and $\mu$ is the mean value of $\sigma$.
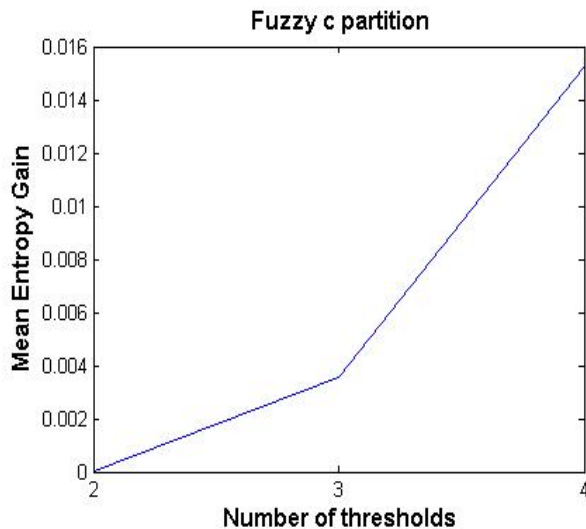


**Figure 7:** Mean entropy gain for all images by the proposed method
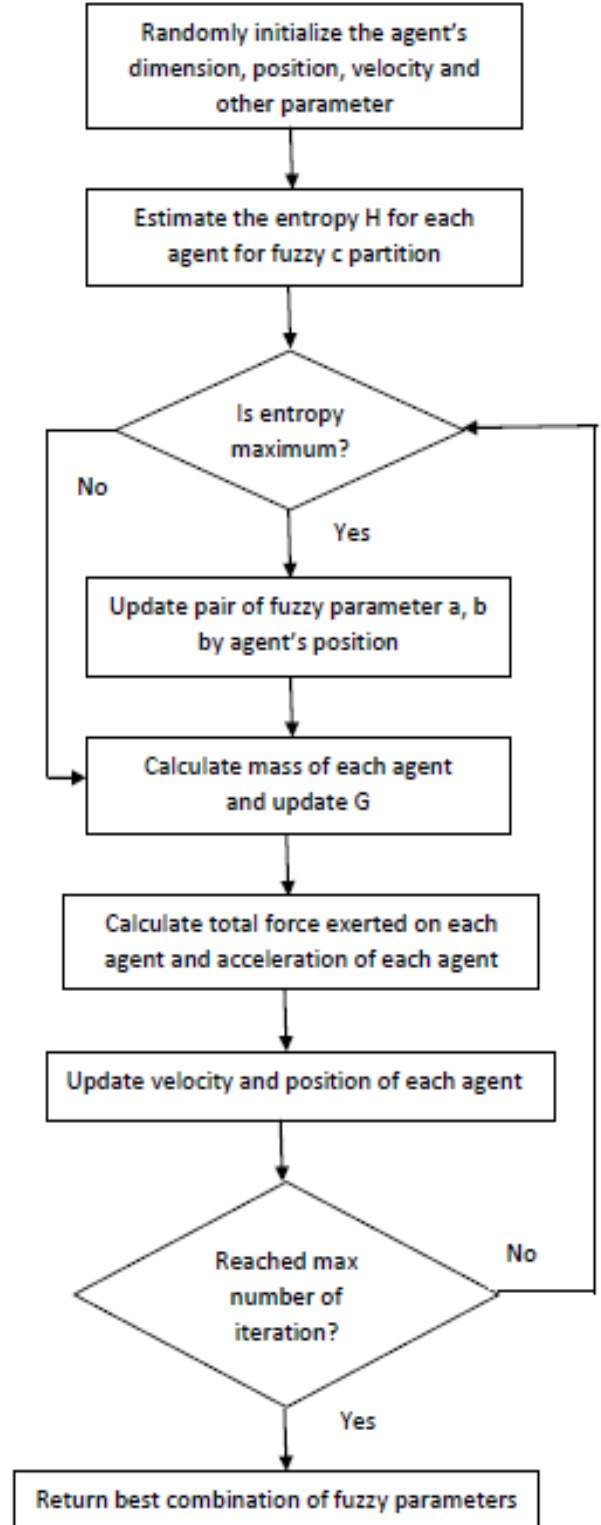


**Figure 8:** Flowchart of proposed method

**Table 1:** Objective function and optimal threshold based on fuzzy entropy criteria

| Image Data Set | c | Fuzzy Entropy | | Optimal Threshold | |
|---|---|---|---|---|---|
| | | PSO | GSA | PSO | GSA |
| Lena | 2 | 0.69314703 | **0.69314718** | 125 | 125 |
| | 3 | 1.09830960 | **1.09861073** | 103,148 | 103,149 |
| | 4 | 1.38117067 | **1.38628154** | 85,128,158 | 87, 129, 159 |
| Pepper | 2 | 0.69314697 | **0.69314718** | 122 | 122 |
| | 3 | 1.09849702 | **1.09861065** | 93, 153 | 93,154 |
| | 4 | 1.38496024 | **1.38628631** | 81, 120, 166 | 80, 121,166 |
| 12003 | 2 | 0.69314663 | **0.69314718** | 107 | 107 |
| | 3 | 1.09807945 | **1.09861025** | 82,130 | 82,131 |
| | 4 | 1.38354637 | **1.38628110** | 67, 106, 145 | 68, 106, 146 |
| 23084 | 2 | 0.69314690 | **0.69314718** | 91 | 92 |
| | 3 | 1.09843110 | **1.09861152** | 60,121 | 60,122 |
| | 4 | 1.38547345 | **1.38628863** | 45, 87, 151 | 45, 87, 152 |
| 376001 | 2 | 0.69314669 | **0.69314718** | 90 | 98 |
| | 3 | 1.09847314 | **1.09861155** | 65 , 132 | 64, 132 |
| | 4 | 1.38451488 | **1.38628943** | 45, 102 , 147 | 44, 102, 148 |
| 113016 | 2 | 0.69314669 | **0.69314718** | 147 | 147 |
| | 3 | 1.09767187 | **1.09860868** | 134, 164 | 133,163 |
| | 4 | 1.38405567 | **1.38627881** | 123, 148, 175 | 122, 148 , 175 |

Both the optimization techniques PSO and GSA were operated in their standard version. The size of population in PSO and in GSA algorithm was set to 20. For fair comparison, both algorithms were executed for the same number of iterations i.e. 50 to make the number of function evaluations same as that of the PSO algorithm. The number of thresholds calculated in this paper was 1-3.

GSA and PSO both are stochastic metaheuristic i.e. random in nature. So, the experiments were repeated to 30 runs for each value and for each fuzzy partition. For PSO, other control parameters w_max =0.4, w_min=0.1, c1= c2=2 were used as defined in [12] and for GSA control parameters are defined in the Section 4. Table 1 provides the maximum fuzzy entropy evaluated for the entire test images and the value of the optimal thresholds corresponding to best fuzzy entropy by both the algorithms.

From the experiments, it has been examined that the GSA produces higher fuzzy c entropy than the results obtained by the PSO for 2-4 level thresholding. In the experiment, fuzzy entropy is measured in eight digit floating point number as fuzzy entropy has less range as compared to other entropies. To investigate the stability of both of the optimization technique and visual quality of segmented image, standard deviation and PSNR respectively are used and presented in Table 3 for test images.

A higher value of PSNR indicates a better quality of thresholding. For all the test images, the proposed method proves to be better than PSO. Results of stan-

dard deviation in Table 3 shows that GSA has better stability in comparison to PSO. The standard deviation of execution time for an image repeated for 30 runs is given in Table 3. Table 2 shows the computation time of proposed method and it has been observed that the computation time of GSA is less than the PSO algorithm.

**Table 2:** Comparison of computation time in sec

| Image Data Set | c | Fuzzy Entropy | |
|---|---|---|---|
| | | PSO | GSA |
| Lena | 2 | 0.68345472 | **0.59643382** |
| | 3 | 0.76930058 | **0.67028742** |
| | 4 | 0.85096460 | **0.72960360** |
| Pepper | 2 | 0.68629233 | **0.59577639** |
| | 3 | 0.76111523 | **0.68016763** |
| | 4 | 0.81905441 | **0.73616665** |
| 12003 | 2 | 0.62258149 | **0.50936502** |
| | 3 | 0.68490198 | **0.58088751** |
| | 4 | 0.76438829 | **0.63904083** |
| 23084 | 2 | 0.61475108 | **0.52676705** |
| | 3 | 0.69071253 | **0.58670343** |
| | 4 | 0.76737386 | **0.65496959** |
| 376001 | 2 | 0.61662361 | **0.51257462** |
| | 3 | 0.68465640 | **0.59372851** |
| | 4 | 0.75321417 | **0.63898595** |
| 113016 | 2 | 0.60287077 | **0.50812043** |
| | 3 | 0.68024741 | **0.57375758** |
| | 4 | 0.71011998 | **0.63280131** |

Figure 7 yields the mean entropy gain by the proposed method for 2-4 level thresholding i.e. it represents the difference between the mean entropy for all images at each level of thresholding. These results show that GSA outperforms for multilevel thresholding.

**Table 3:** PSNR and standard deviation of entropy that yields best result

| Image Data Set | c | PSNR (dB) | | Standard Deviation | |
|---|---|---|---|---|---|
| | | PSO | GSA | PSO | GSA |
| Lena | 2 | **10.11505776** | **10.11505776** | 0.00000054 | **0.00000001** |
| | 3 | 13.25998845 | **13.38324736** | 0.00046983 | **0.00000173** |
| | 4 | 15.60343340 | **15.76010709** | 0.00398558 | **0.00000864** |
| Pepper | 2 | **10.48588785** | **10.48588785** | 0.00000068 | **0.00000000** |
| | 3 | 13.12702554 | **13.16831907** | 0.00058510 | **0.00000161** |
| | 4 | 15.19555078 | **15.20680883** | 0.00154086 | **0.00000459** |
| 12003 | 2 | **9.69609508** | **9.69609508** | 0.00000115 | **0.00000001** |
| | 3 | 12.66514231 | **12.77397929** | 0.00058510 | **0.00000256** |
| | 4 | 15.20168031 | **15.32594882** | 0.00256040 | **0.00001187** |
| 23084 | 2 | 9.67270633 | **9.73414060** | 0.00000068 | **0.00000000** |
| | 3 | 13.25866797 | **13.32598105** | 0.00018559 | **0.00000067** |
| | 4 | 15.59120966 | **15.59426411** | 0.00064487 | **0.00000407** |
| 376001 | 2 | 8.71728776 | **9.01966110** | 0.00000077 | **0.00000001** |
| | 3 | 12.48230636 | **12.48959956** | 0.00012982 | **0.00000067** |
| | 4 | 14.80698029 | **14.90069306** | 0.00133660 | **0.00000333** |
| 113016 | 2 | **11.57386929** | **11.57386929** | 0.00000077 | **0.00000001** |
| | 3 | 14.53615103 | **14.60635305** | 0.00105359 | **0.00000267** |
| | 4 | 16.58813899 | **16.63353077** | 0.00173955 | **0.00001675** |

## 6  Conclusion

In this paper, a novel thresholding method based on fuzzy entropy using gravitational search algorithm has been proposed. The performance of the proposed method has been evaluated on the six test images and compared with particle swarm optimization on various parameters such as maximum entropy, standard deviation of entropy, PSNR. In proposed method entropy has been used as an objective function. We performed the experiment for bilevel and extended to multilevel thresholding. The results obtained from test images demonstrated that GSA performs better than PSO in terms of entropy, PSNR, stability and computation time. The experimental results show the effectiveness of GSA.

GSA has better characteristics in searching optimal fuzzy parameters to calculate threshold. The other advantage of the GSA is that it uses few control parameters than PSO. In GSA, at every iteration, fitness evaluation is carried out by best and worst fitness of population that yields better and efficient results. For future scope, 2D fuzzy entropy may also used in combination with multilevel thresholding for image segmentation.

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